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RANSAC versus CS-RANSAC

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Abstract

A homography matrix is used in computer vision field to solve the correspondence problem between a pair of stereo images. RANSAC algorithm is often used to calculate the homography matrix by randomly selecting a set of features iteratively. CS-RANSAC algorithm in this paper converts RANSAC algorithm into two-layers. The first layer is addressing sampling problem which we can describe our knowledge about degenerate features by mean of Constraint Satisfaction Problems (CSP). By dividing the input image into a $N \times N$ grid and making feature points into discrete domains, we can model the image into the CSP model to efficiently filter out degenerate features. By expressing the knowledge about degenerate feature samples using CSP in the first layer, so that computer has knowledge about how to skip computing the homography matrix in the model estimation step for the second layer. The experimental results show that the proposed CS-RANSAC algorithm can outperform the most of variants of RANSAC without sacrificing its execution time.

Introduction

In recent years, augmented reality (AR) systems have received considerable attention as an effective method for visualizing virtual information represented in text, still images, animations, videos, or 3D objects annotated on cameras or mobile device displays. Some studies have explored how to use an AR system to perform highly complex and sophisticated tasks, such as aircraft maintenance and repair (Jo et al. 2014; Crescenzio et al. 2011; Lee, Rosli and Jo. 2012). However, to correctly annotate a virtual object on a camera image in an AR system, it is necessary to accurately estimate the pose of the virtual object, which can be done by calculating the homography matrix between reference and camera images.

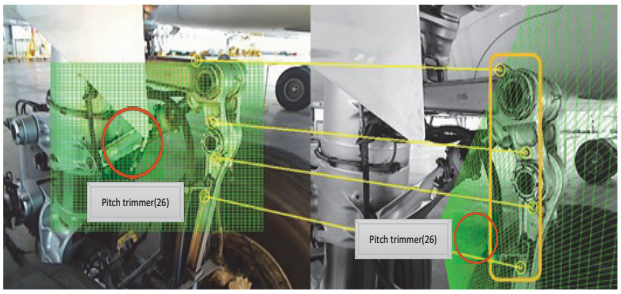
The calculation of a homography matrix can be solved using the RANSAC algorithm, where each sample set is randomly selected. The accuracy of the homography matrix generated here depends heavily on the selected random features. However, RANSAC algorithm picks the features without considering their locations, which may lead to the selection of sets of features distributed linearly or too close to one another, resulting in an inaccurate homography matrix.

Figure 1 illustrates the pixels mapping (shown as green grid) from the reference images in left column to the camera images in right column. The pixels mapping is done using a homography matrix calculated based on four selected pairs of features (marked as yellow circles). In figure 1(A), the selected features lie on the same linear sequence, causing the homography matrix to be accurate only for estimating objects in some areas where the features are selected (i.e. area inside the yellow rounded rectangle), but fail in estimating the pose of other objects in other areas (e.g. the pitch trimmer annotated with orange circles). The same result happens when the selected features lie too close to one another, as shown in figure 1(B). In contrast, as shown in figure 1(C), a desirable homography matrix can be obtained to estimate the whole area of the image by selecting only well-distributed features.

As illustrated in figure 1, accurately estimating the whole area is more important than estimating only a specific area of an image to avoid the failure of the tracking system to track the target object as a result of some factors such as fast movements or occlusions, particularly in an AR system requiring a robust tracking system. To overcome the problems described in the figure 1 and also to improve the existing variants of RANSAC, we are here proposing CS-RANSAC algorithm which embed CSP into the classic RANSAC.

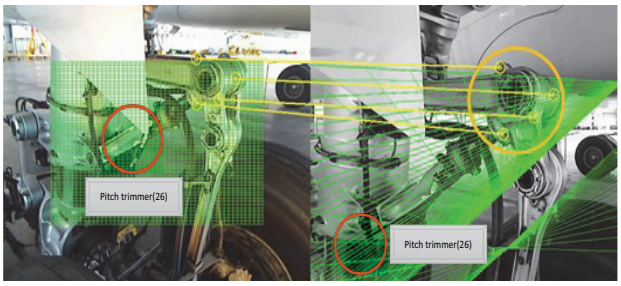
RANSAC Algorithm

RANSAC algorithm basically consists of two steps: hypothesis and evaluation (Zuliani 2012). In hypothesis step, a small subset of data is randomly selected to compute a model to fit the dataset. In evaluation step, this model is us-



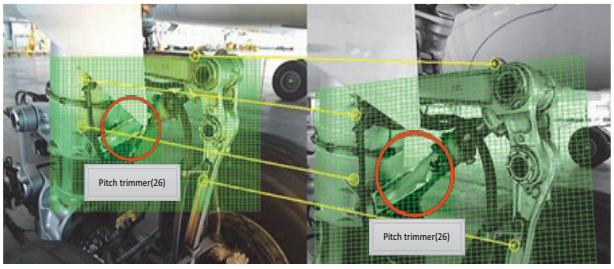
Reference image Camera image

(A) Collinear features set



Reference image Camera image

(B) Adjacent features set



Reference image Camera image

(C) Well-distributed features set

Figure 1: Homography matrix calculation by RANSAC algorithm using different features sets.

ed to determine a consensus set by checking which data from the dataset are consistent with the model (i.e. the number of inliers). These steps are iteratively repeated until the algorithm fails to find a better model to represent the dataset.

Many studies have been conducted to improve the classic RANSAC algorithm for the calculation of a homography matrix. Some studies focused on finding the true set of inliers, whereas some others on the sampling step by filtering degenerate samples according to some conditions. LO-RS algorithm (Chum, Matas, and Kittler 2003) improves the inlier rate of RANSAC algorithm by locally optimizing an area of the image that is highly condensed by features. Here, another RANSAC algorithm is run within each iteration of the RANSAC algorithm. T-RS algorithm (Vincent and Laganiere 2001) computes the areas of four triangles determined by the random features and retains only those whose areas are greater than a given threshold. The threshold value is set to a generous number at the first iteration and is tightened once the homography matrix becomes stable. MFF-RS algorithm (Wu and Fang 2007) applies two filters in the sampling step: angle and length filters. Each sample feature is considered as a vector and is retained as an inlier only if its value is lower than the median flow value derived by comparing all the random features. All these techniques have successfully improved the classic RANSAC algorithm for calculating a data model, particularly the homography matrix of an image pair. However, the resulting homography matrix may not be able to accurately estimate the pose of objects in the whole image because they could not express which feature samples are degenerate samples with the efficient computational model.

CSP for RANSAC

Most of artificial intelligence (AI) problems can be formulated as a constraint satisfaction problem (CSP) (Russell, and Norving 2010) defined by a set of variables $X_1, X_2, \dots, X_n$ and a set of constraints $C_1, C_2, \dots, C_m$, where each variable X_i has a nonempty domain D_i of some possible values v_i. The state of a problem is defined by an assignment of values to some or all of the variables $\{X_i = v_i, X_j = v_j, \dots\}$. An assignment is said to be consistent if it does not violate any constraints, and it is said to be complete if every variable is mentioned.

A solution to the CSP is then the complete assignment that satisfies all the constraints (Russell, and Norving 2010). Treating a problem as a CSP confers two important benefits in vision problems. One is to represent the knowledge in a form of constraints, the other is to develop effective and generic heuristics requiring no additional domain-specific knowledge.

This paper proposes CS-RANSAC algorithm by introducing CSP into a computer vision problem of model estimation, particularly a homography matrix. The sampling problem of RANSAC algorithm is modeled as a CSP model to help filter out degenerate configurations in this model (i.e. sets of collinear or adjacent features in the estimation of a homography matrix), and therefore, gain a homography matrix with higher accuracy to estimate the whole area of an image.

With the sampling problem of a RANSAC algorithm being considered as a CSP, the variable is defined as a set of feature points used to calculate the data model, denoted as f_k with $k \in \{1,2, \dots, n\}$ and n is the number of samples. To define the domain and value of each variable, particularly in a model estimation problem for two corresponding images, the input image I_s is first divided into an $N \times N$ grid to allow the CSP model to efficiently check the constraints of each feature pair. The value assignable to each

feature is defined as the location of the corresponding cell where the feature lies in, which can be represented by a two tuple (row, col). Therefore, features lie in the same cell will be mapped into a same value of a variable. With this, many degenerate features lying too close to one another can be eliminated. The domain of each feature f_k is a set of any possible locations of the corresponding cell in the grid. Together with the constraints defined in next section, this $N \times N$ grid helps to reduce the search scope, such that only one sample can be selected in a cell, and thus, more time can be spent for some adjacent cells which satisfy constraints.

Constraints for expressing knowledge about how to filter out degenerate features

As shown in Table 1, the relationship between all the selected random features is defined using a set of constraints $C_{f_{n-1}f_n}$. In this paper, we employ a set of linear and distance constraints to filter the degenerate samples in a homography matrix calculation problem to remedy the problems described in the figure 1. By applying these constraints to Images from UKBench dataset (Nister and Stewenius 2006), we also would like to verify that CSP-based filtering degenerate features samples generally outperforms other variants of RANSAC algorithm. Linear constraints ensure that no selected samples lie on the same linear sequence, whereas distance constraints ensure that all samples lie far enough from one another.

Table 1: Features relationships in CS-RANSAC Algorithm

	f_1	f_2	f_3	$\cdots$	f_n
f_1		$C_{f_1f_2}$	$C_{f_1f_3}$	$\cdots$	$C_{f_1f_n}$
f_2			$C_{f_2f_3}$	$\cdots$	$C_{f_2f_n}$
f_3				$\cdots$	$C_{f_3f_n}$
$\vdots$					$\vdots$
				$\cdots$	$C_{f_{n-1}f_n}$
f_n					

Let $C_{f_if_j}$ be a set of constraints between two features f_i and f_j. $C_{f_if_j}$ is true when both linear and distance constraints are satisfied. With row_i and row_j are the row indices, and col_i and col_j are the column indices of the corresponding cells of features f_i and f_j, respectively, the linear and distance constraints are defined as follows.

Definition 1 (Linear constraints) *A set of* n *features satisfies linear constraints iff*:

$i, j \in \{0, \dots, n\} \Rightarrow$

$<row_i \neq row_j>$ *is true and*

$<col_i \neq col_j>$ *is true and*

$<col_j \neq col_i \pm |row_i - row_j|>$ *is true*

Definition 2 (Distance constraints) *A set of* n *features satisfies distance constraints iff*:

$i, j \in \{0, \dots, n\} \Rightarrow$

$<[|row_i - row_j| = 1]$ *and* $[|col_i - col_j| > 2]>$ *is true and*

$<[|row_i - row_j| = 2]$ *and* $[|col_i - col_j| > 1]>$ *is true*

CS-RANSAC Algorithm

Generally, RANSAC algorithm can be divided into two main steps: the sampling step and the model estimation step. CS-RANSAC algorithm modifies the sampling step of the RANSAC algorithm, where it incorporates a CSP model to filter out degenerate samples before they can be used in the model estimation step. As outlined in

Algorithm 1 CS-RANSAC.

Input: Extracted features from the input image, the maximum number of sampling iterations θ_s, the error threshold θ_ε.

Initialize H = NULL, $bestInliers$ = 0, $nIterations$ = 1, $nSimSampling$ = 0, i = 0.
Repeat
  Initialize f_n = NULL, f_k = NULL, $nInliers$ = 0, $sIterations_i$ = 0, H_i = NULL
  While f_n is empty or f_n is not consistent with the constraints in Definitions 1 and 2 **do**
    If $sIterations_i < \theta_s$ **do**
      Sample n features f_n randomly
      $sIterations_i = sIterations_i + 1$
      Check the consistency of f_n according to constraints in Definitions 1 and 2
    Else
      Sample n features f_n based on similarity rankings
      $nSimSampling = nSimSampling$ +1
      Exit While
    End If
  End While
  $f_k = f_n$
  Calculate H_i based on f_k
  Calculate $nInliers$ according to θ_ε
  If $nInliers > bestInliers$ **do**
    $H = H_i$
    $bestInliers = nInliers$
  End If
  Calculate $nIterations$ using Eqn. 1
  $i = i + 1$
Until $i = nIterations$

Output: H

Algorithm 1, n features are first selected randomly from the extracted features of the input image I_s. The sampled features are then checked for their consistency by the constraints set defined in the previous section. They are considered for the next step only if all the constraints are satisfied. If any pair of features fails to satisfy any constraint, then CS-RANSAC first checks the sampling iteration denoted as $sIterations$. If $sIterations$ exceeds the predefined threshold θ_s, then CS-RANSAC instead samples another set of n features with the highest similarity ranking based on the Euclidean distance. Here, θ_s is set to 100 sampling iterations to avoid the algorithm to loop forever, which have not happened or have not even reached to near that point yet in our real implementation. The rest of the CS-RANSAC algorithm is similar to that of the classic RANSAC algorithm, in which the data model is iteratively calculated and updated using a set of sampled features derived from the previous step (i.e. either the features set selected by the CSP model or the similarity ranking). The data model is evaluated based on the number of inliers (i.e. the number of feature points agrees with the data model generated in current iteration with respect to an error threshold θ_ε).

The number of iterations of the CS-RANSAC algorithm is updated at the end of each iteration as follows:

$$nIterations = \left\lceil \frac{\log(1-p)}{\log(1-(1-e)^n)} \right\rceil \quad (1)$$

where p is the probability of all sampled features being inliers, and is set to 0.9999 to obtain high accuracy. Conversely, e is the probability of at least one sampled feature being an outlier, calculated using Eq. 2. Finally, n is number of features used to calculate a data model.

$$e = 1 - \frac{Number\ of\ Inliers}{Total\ Number\ of\ features} \quad (2)$$

The data model with the largest number of inliers is retuned as the best data model (i.e. homography matrix $\boldsymbol{H}$) which corresponds all feature pairs between the source image $\boldsymbol{I_s}$ and the destination image $\boldsymbol{I_d}$.

Experiments

Images from UKBench dataset (Nister and Stewenius 2006) with a size of 640×480 are used for evaluation. A ground truth tool is built to check the validity of the matching results of the SURF features (Bay et al. 2008) based on the Euclidean distance. After extracting SURF features from all test images (i.e. both source and destination images), the similarity between extracted features from each image pair are calculated by the Euclidean distance to generate pairs of matched feature points, which are then sorted and stored to be used when the proposed CSP model fails to sample a set of n constraints satisfying feature points. Finally, assuming that all feature points lie in a real-world plane, the homography matrix for each image pair is calculated.

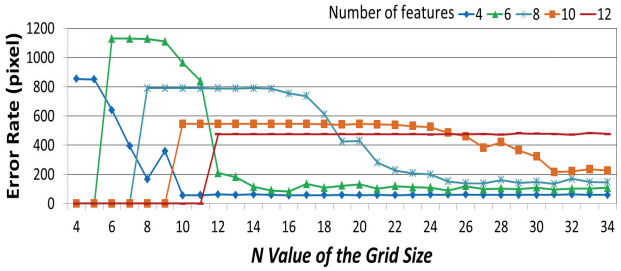


Figure 2: Error rates for various grid sizes and numbers of features.

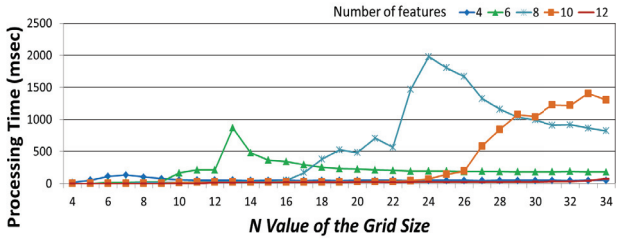


Figure 3: Processing times for various grid sizes and numbers of features.

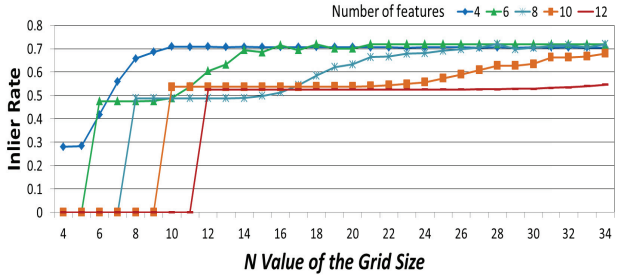


Figure 4: Inlier rates for various grid sizes and numbers of features.

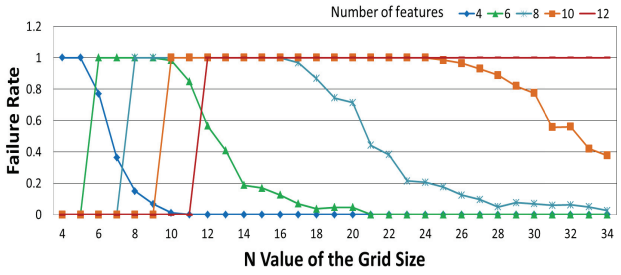


Figure 5: Failure rates for various grid sizes and numbers of features.

Table 2: Comparison of several variants of RANSAC algorithm for the processing time (ms).

	G1		G2		G3		G4		G5	
	Avg.	Std.	Avg.	Std.	Avg.	Std.	Avg.	Std.	Avg.	Std.
RS	79.92	36.11	61.15	35.05	53.52	36.38	67.69	33.81	11.29	11.54
T-RS	80.96	38.17	67.53	35.75	51.34	34.15	65.60	27.17	10.45	10.22
MFF-RS	89.71	38.88	66.75	36.44	69.33	36.11	79.77	37.42	11.79	12.96
LO-RS	**58.34**	27.74	65.29	33.80	44.65	23.21	**52.13**	17.81	13.46	10.33
CS-RS	65.47	**23.12**	**52.46**	**22.39**	**41.43**	**21.37**	55.33	**14.74**	**9.76**	**8.25**

Table 3: Comparison of several variants of RANSAC algorithm for the error rate (pixel).

	G1		G2		G3		G4		G5	
	Avg.	Std.	Avg.	Std.	Avg.	Std.	Avg.	Std.	Avg.	Std.
RS	81.57	36.89	35.59	11.62	48.01	22.50	128.99	97.98	41.93	29.80
T-RS	88.71	104.58	37.28	33.11	48.48	28.56	124.82	76.07	39.12	32.96
MFF-RS	99.84	94.69	37.99	14.03	49.00	23.66	157.78	156.88	44.54	43.99
LO-RS	189.74	358.79	217.02	495.21	295.52	845.65	367.95	617.67	227.66	328.79
CS-RS	**75.12**	**26.620**	**34.51**	**10.68**	**44.63**	**20.48**	**105.13**	**46.80**	**32.30**	**20.59**

Table 4: Comparison of several variants of RANSAC algorithm for the number of inlier.

	G1		G2		G3		G4		G5	
	Avg.	Std.	Avg.	Std.	Avg.	Std.	Avg.	Std.	Avg.	Std.
RS	367.04	15.04	2779.16	33.77	1829.35	24.18	272.95	10.00	1637.34	26.66
T-RS	368.33	14.43	2782.08	32.17	1828.94	22.61	272.19	9.83	1641.83	26.18
MFF-RS	356.22	15.56	2755.89	68.31	1806.23	31.80	261.88	14.11	1625.64	38.21
LO-RS	**381.23**	14.01	**2806.44**	**14.85**	**1854.40**	**13.65**	**280.45**	**6.05**	**1660.93**	**12.55**
CS-RS	371.29	**13.10**	2788.31	24.12	1837.43	17.00	277.56	6.26	1651.10	18.76
GT	434.58	91.13	2963.75	452.31	1884.14	438.95	306.83	61.83	1709.22	231.93

All matching features are manually verified, one feature at a time. If there is any pair of features classified as an inlier (i.e. a pair of corresponding features by the Euclidean distance) but found to be an outlier (i.e. a pair of feature points that does not match with each other according to human vision), then it is manually classified as an outlier. A total of four measures are used to evaluate the proposed algorithm: error rate, processing time, inlier rate, and failure rate. Error rate refers to the accuracy of the homography matrix, which is derived using the symmetric transfer error (Hartley and Zisserman 2004) as follows:

$$Error\ Rate = \sum_{k} d(p_k, H^{-1}p'_k)^2 + d(p'_k, Hp_k)^2 \qquad (3)$$

where d is the Euclidean distance between two feature points. p and p' are any pixel in the source image and its estimation in the destination image according to H, respectively. A P×P grid is overlaid onto the source image, and p is randomly selected from each cell of the grid.

Processing time is the time required by each algorithm to generate a homography matrix H for each image pair, measured on Intel Core I5 CPU 1.80GHz. Inlier rate indicates the probability of features agreeing with H.

$$Inlier\ Rate = \frac{Number\ of\ inliers}{Number\ of\ features} \qquad (4)$$

Finally, failure rate refers to the rate at which the proposed CSP model fails to find a set of n constraints satisfying feature points within 100 sampling iterations.

$$Failure\ Rate = \frac{nSimSampling}{nIterations} \qquad (5)$$

where $nSimSampling$ is the number of iterations which use top n features by similarity to calculate a homography matrix, and $nIterations$ is the number of iterations required by CS-RANSAC to generate H for each image pair.

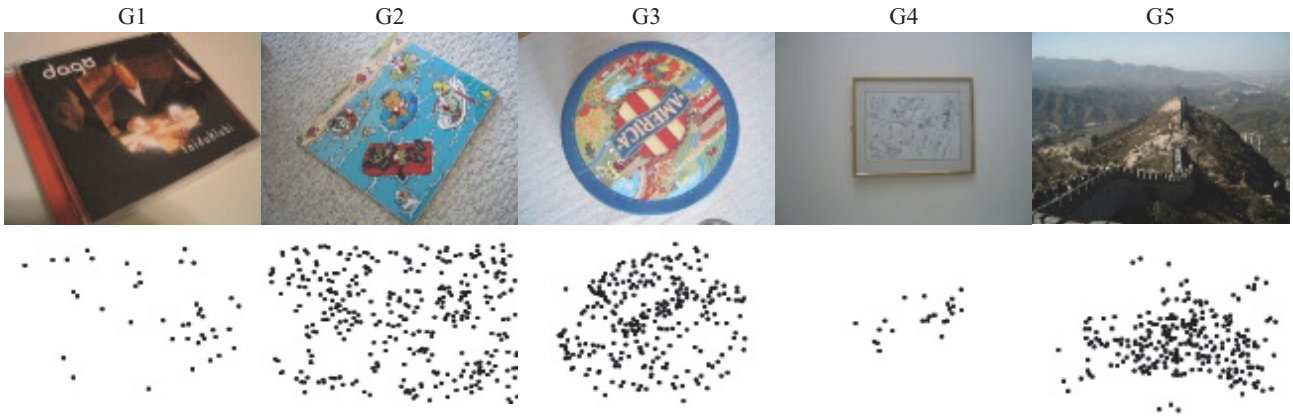


Figure 6: The classification of test images based on feature distributions (G1: few features distributed over the whole image; G2: many features distributed over the whole image; G3: many features distributed at the center of the image; G4: few features distributed over a specific area in the image; G5: many features distributed over a specific area in the image).

Results and Discussion

Grid Size and Number of Samples for each Image

Though four-point algorithm (Hartley and Zisserman 2004) has suggested to use sets of four matched points to estimate a planar homography, this experiment is conducted to determine whether the grid size affects the suggestion of four-point algorithm. CS-RANSAC algorithm is executed to calculate a homography matrix H for each image pair. This experiment is conducted on various sizes of grid and samples using 33 pairs of images, to determine the optimal grid size and the optimal number of samples for each grid size in the proposed algorithm. Its performance is evaluated by the error rate, the processing time, the inlier rate, and the failure rate. The average value of each measure is calculated and presented in graphs, as shown in figures 2, 3, 4, and 5, which clearly show that using four samples indeed is also the optimal number of features in CS-RANSAC. Compared to using a larger number of samples, it produces the lowest error and failure rates, requires the shortest processing time, and generates the highest inlier rate.

For the grid size, as shown in figures 2 and 5, the error and failure rates decrease sharply when the value of N is three to four times the number of samples. This is because when the number of samples is greater than or equal to the value of N, it is not possible to find a set of features that satisfy all the defined constraints. Therefore, it will invoke the alternative sampling method and increase the failure rate. In addition, because the features are selected based only on their similarity rankings (i.e. there is no guarantee that the selected samples are well distributed), it increases the error rate. Similarly, as shown in figure 4, the highest inlier rate is observed when the value of N is three to four times the number of samples, but this comes at the expense of the processing time when the number of samples is larger than four, as shown in figure 3. These results therefore suggest four as the optimal number of samples and 17×17 as the optimal grid size, as shown in Table 2.

Table 5: Algorithm behaviors using four features and grid size of **17×17**.

Error rate	Processing time	Inlier rate	Failure rate
57.19 pixels	46.38 msec	70.71%	0%

CS-RANSAC vs. Other RANSAC Algorithms

In this experiment, five variants of RANSAC algorithm with different basic concepts are compared in terms of processing time, error rate, and number of inliers. In CS-RANSAC algorithm, as described in previous section, all the selected samples have to satisfy the defined constraints to be used in the homography calculation step. In T-RS, the selected samples are filtered based on the area of triangles the features form. This only solves the distance problem, but not the linear problem, which may affect the results. The constraints in MFF-RS rely only on the length or angle of the features which are treated as vectors. Therefore, MFF-RS cannot ensure the distribution of the features. In contrast, LO-RS focuses only on optimizing the inlier rate, by employing only the inlier set found in previous steps to recalculate the homography matrix. With this, the number of inliers found by LO-RS is expected to be the best among five variants of RANSAC algorithm in this experiment. But worth to be noted, the error rate here is calculated based on the pixel mapping from whole area of the image, which is why LO-RS has a very high error rate. A total of 50 images are selected and divided into five groups based on the features distribution pattern (figure 6) such that each

group has 10 pairs of images. Each algorithm is executed to produce a homography matrix for each image pair. Average (Avg.) and standard deviation (Std.) of the error rate, the processing time, and the number of inliers are calculated for evaluation.

As shown in Table 3, CS-RANSAC generally provides better runtime compared to the others, with average reductions ranging from 6% to 20%. This is because only the constraints satisfying random samples are used, so that it can produce a correct homography matrix in each iteration (as shown in Table 4, CS-RANSAC provides lower error rate compared to other algorithms, with 3% to 85% improvement of the error rate), and thus requires lesser amount of time to produce the best homography matrix for each image pair. However, for images in G1 and G4, which have only a few feature points, LO-RS provides better execution time than CS-RANSAC by 6% and 12%, respectively. This is because there are only few features in the whole image area, so that the proposed CSP model has difficulty in finding the constraints satisfying features, which cause the increase of the sampling iteration and therefore, increase the processing time. For LO-RS, as expected, it provides the largest number of inlier (Table 5,) and thus it can accurately estimate only specific parts of the image, reflecting the case shown in figure 1(B).
Because TS-RS does not solve the linear problem, it was found to have lower accuracy (i.e. higher error rate and smaller number of inliers) compared to the CS-RANSAC algorithm. In addition, by dividing the image into a grid, it helps to eliminate many degenerate features in early steps without any further calculation. However, T-RS has to calculate the relationship of each feature before it can eliminate some degenerate ones.

Conclusions and Future Works

The experimental results show that CS-RANSAC generally outperforms four other variants of RANSAC algorithm (i.e. classic RANSAC, T-RS, MFF-RS, and LO-RS) in terms of accuracy and processing time without sacrificing its execution time. The feature distribution by describing in terms of CSP strongly affects both the processing time and accuracy without sacrificing its execution time and inlier rate. Even though LO-RS provides the highest inlier rate, there is no significant difference between LO-RS and CS-RS. However, the proposed CS-RANSAC also shows its superiority in estimating the pose of objects in whole image area, which can solve the problems described in the figure 1. In addition, it is worth noting that the CSP used in CS-RANSAC is more general and robust scheme than any other methodology used in four other variants of RANSAC compared in this research. Through the motivation derived from the figure 1 and the experiment with UKBench dataset (Nister and Stewenius 2006), we proved that the efficient filtering of degenerate features and guiding how to choose the right feature samples using CSP can improve overall performance of RANSAC algorithm.

The future work should be improving the CS-RANSAC algorithm by introducing different constraints for various types of images, in which the grid size can be also adjusted and allow CS-RANSAC to automatically select a set of constraints depending on the distribution pattern of its features.

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