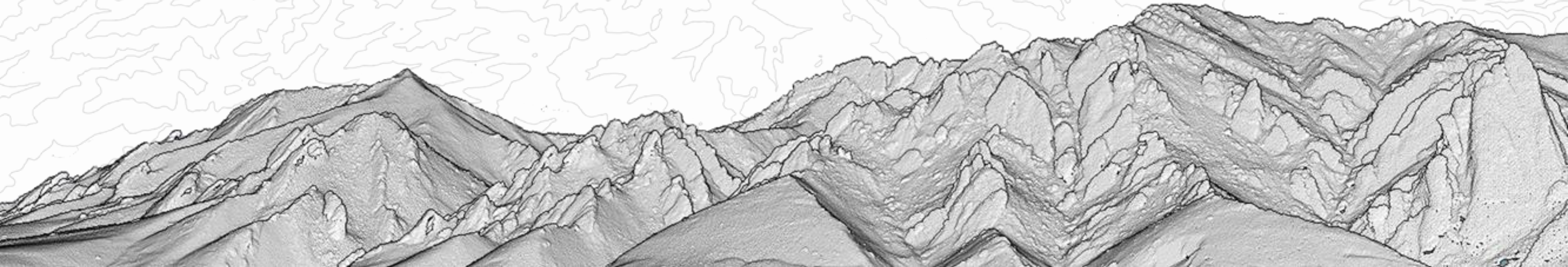
Error analysis in vertical topographic differencing

Cassandra Brigham

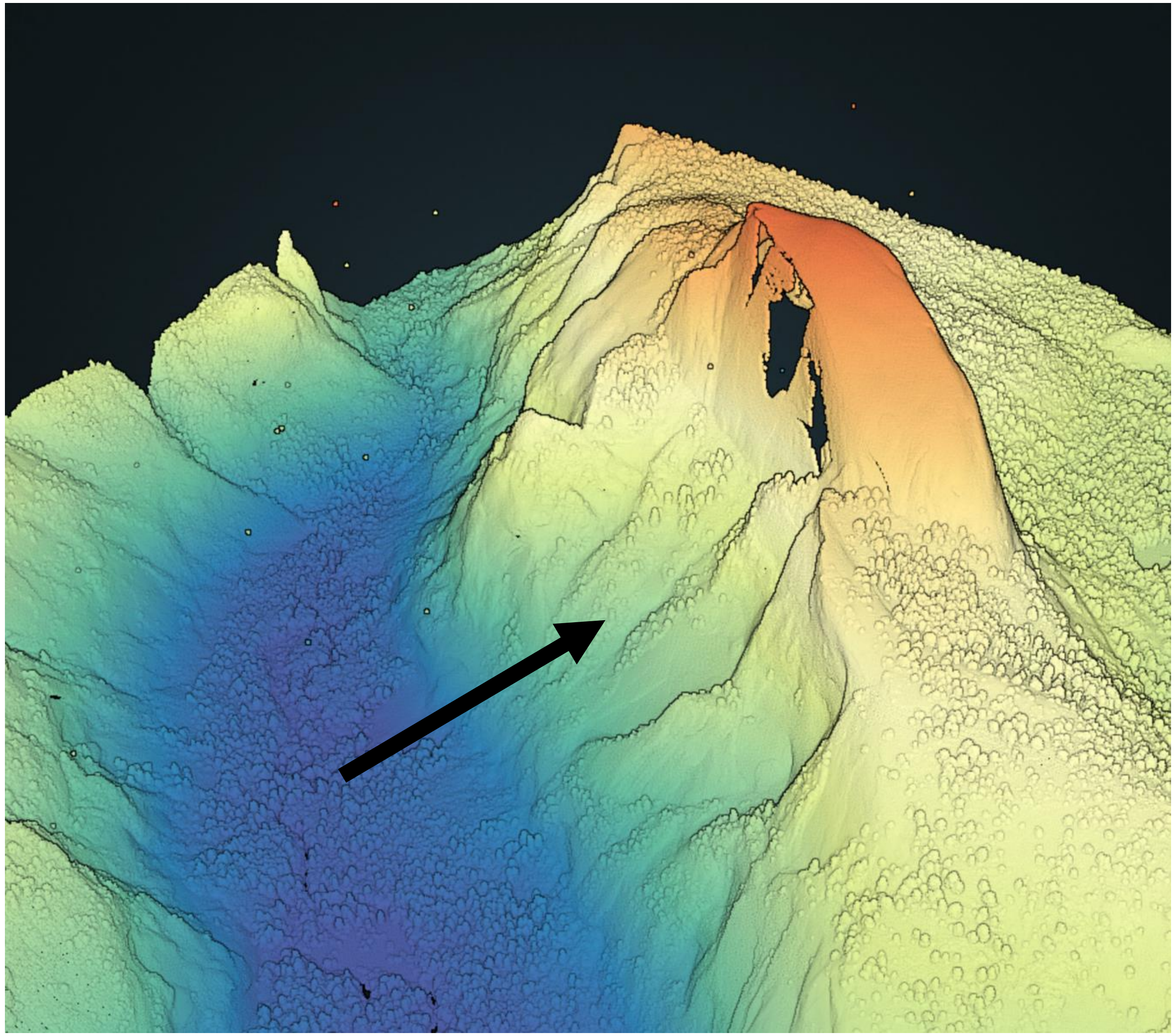
CLaSH Workshop | September 23, 2026

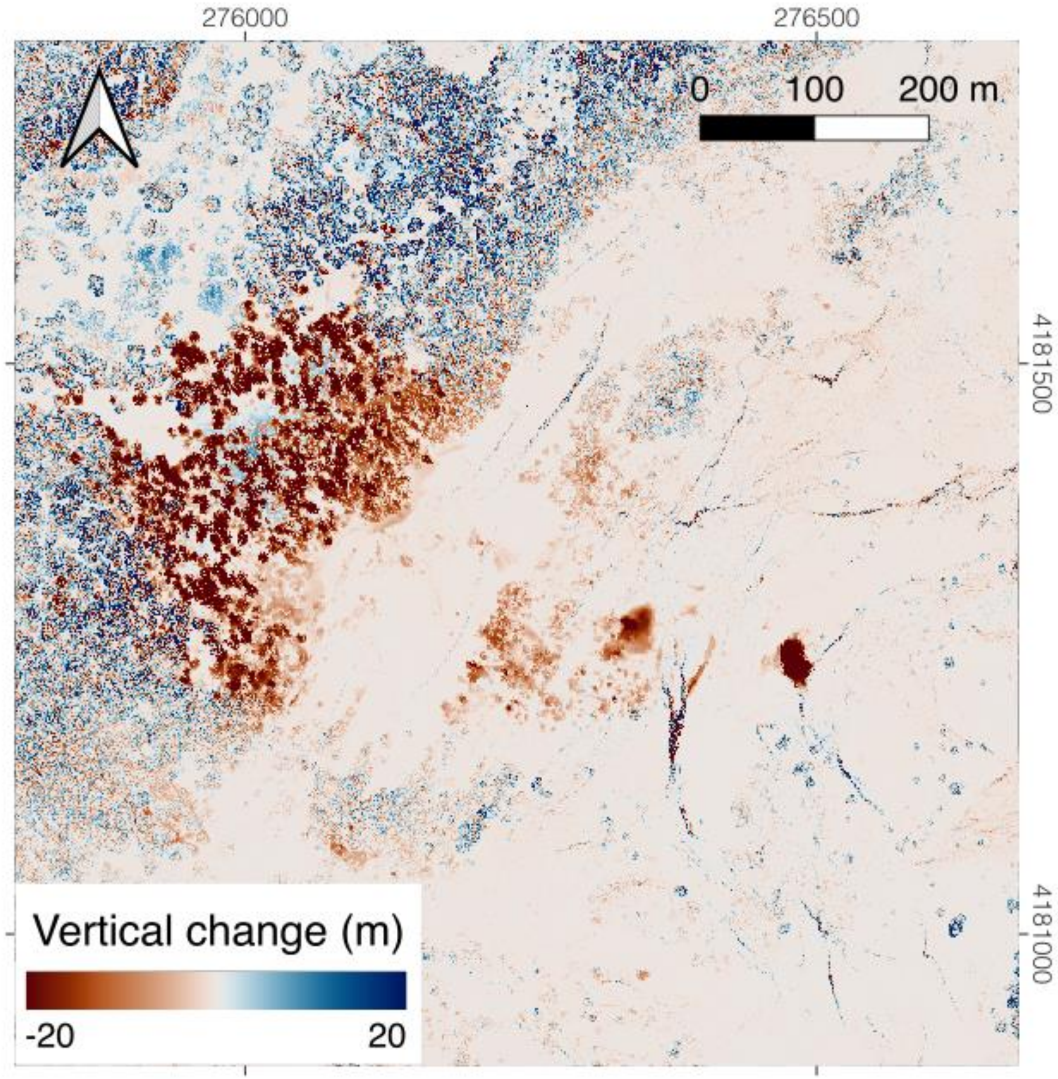


Outline

- Sources of error in lidar differencing
- Theory: variograms and spatial uncertainty
- The *topochange* Python tool
- Los Angeles Eaton Fire application

Error in Vertical Differencing





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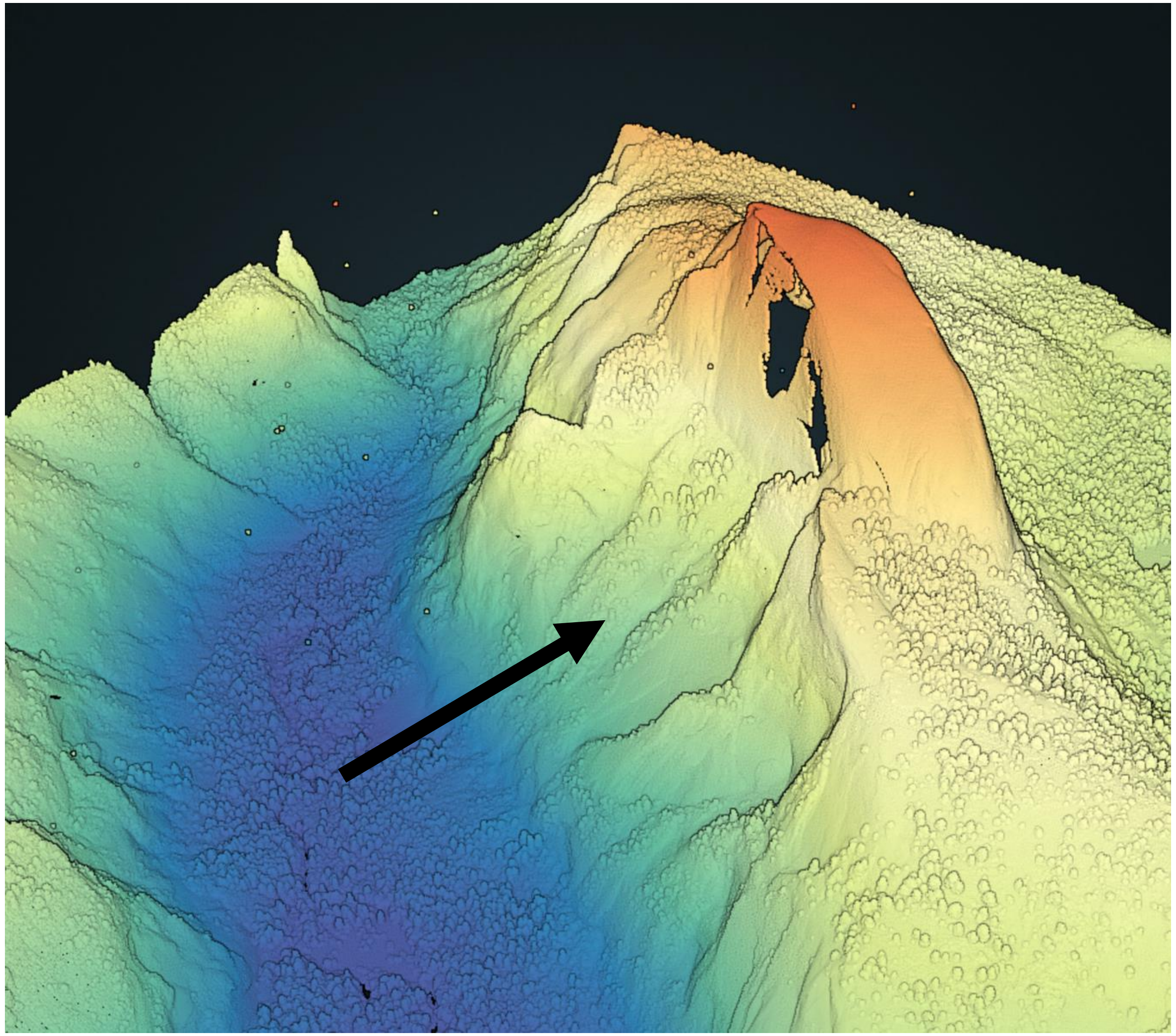

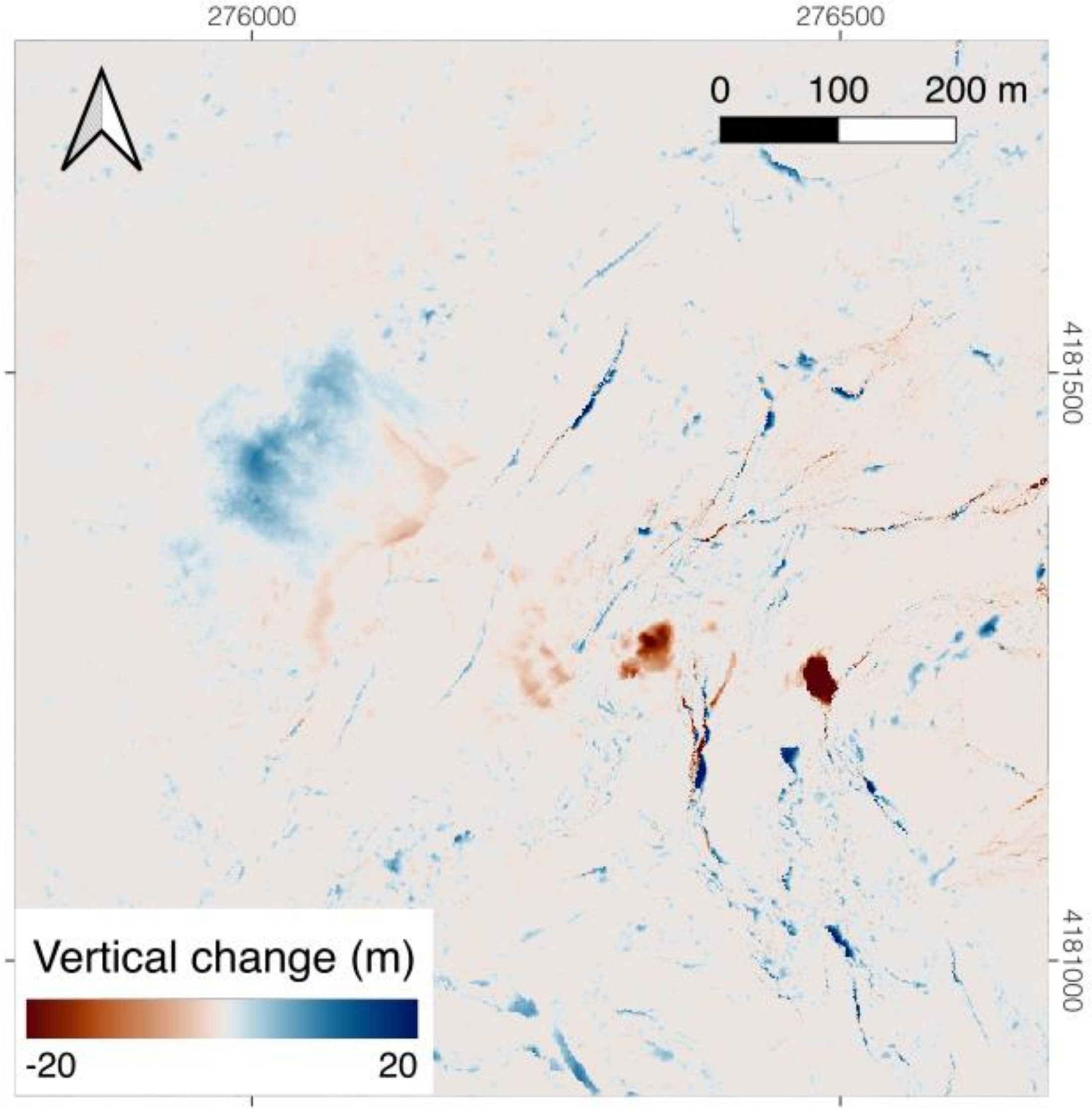

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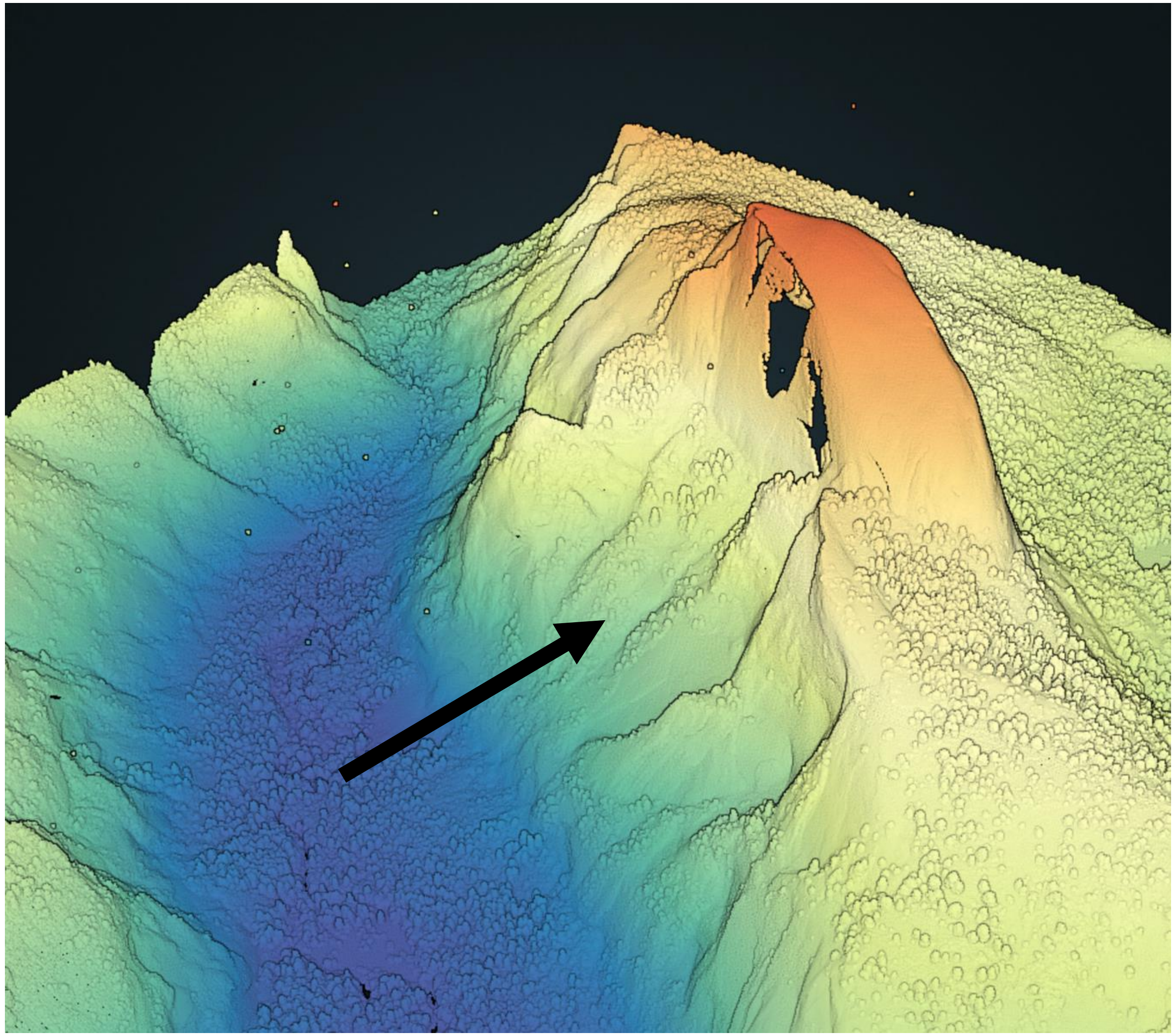
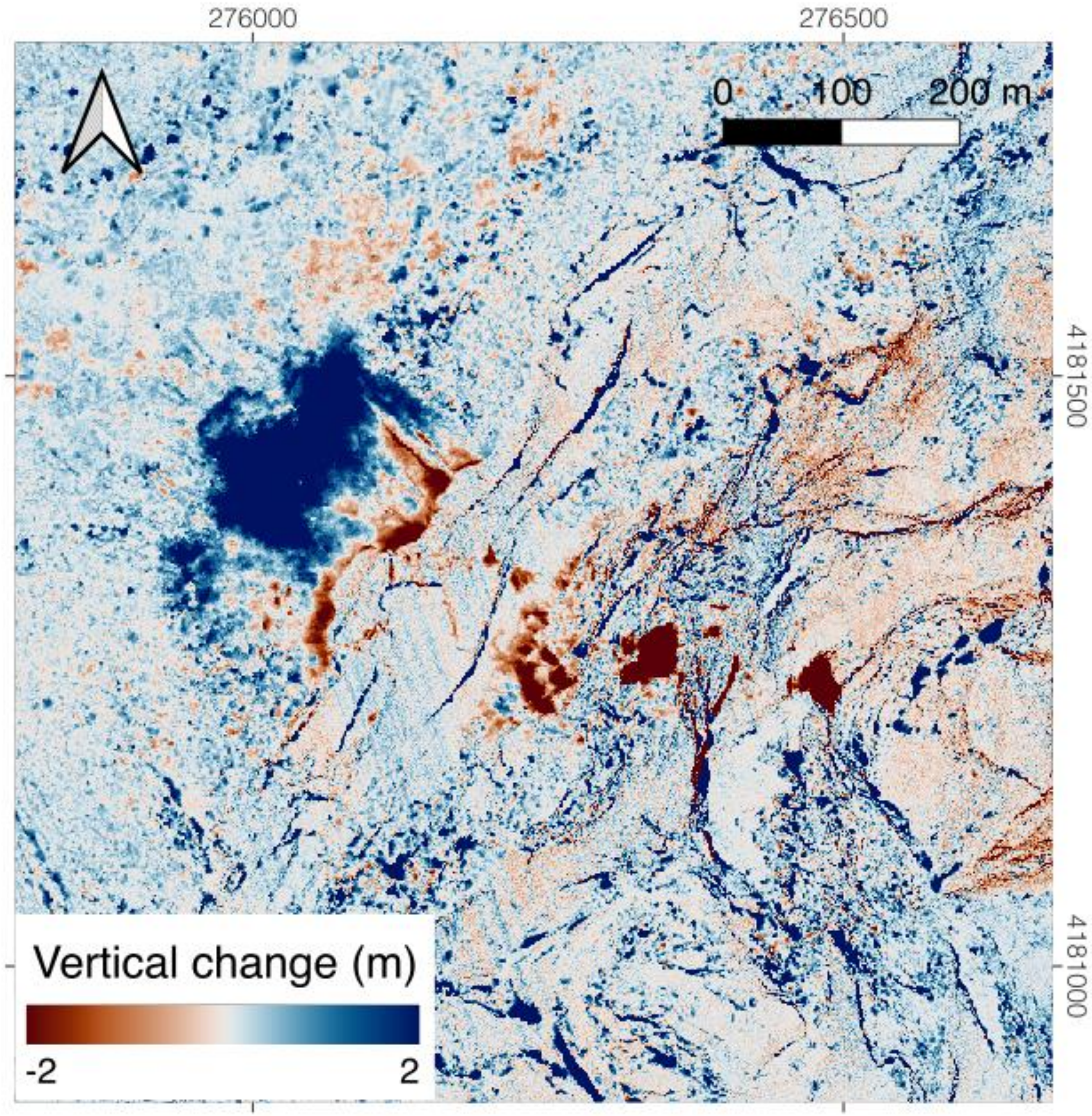
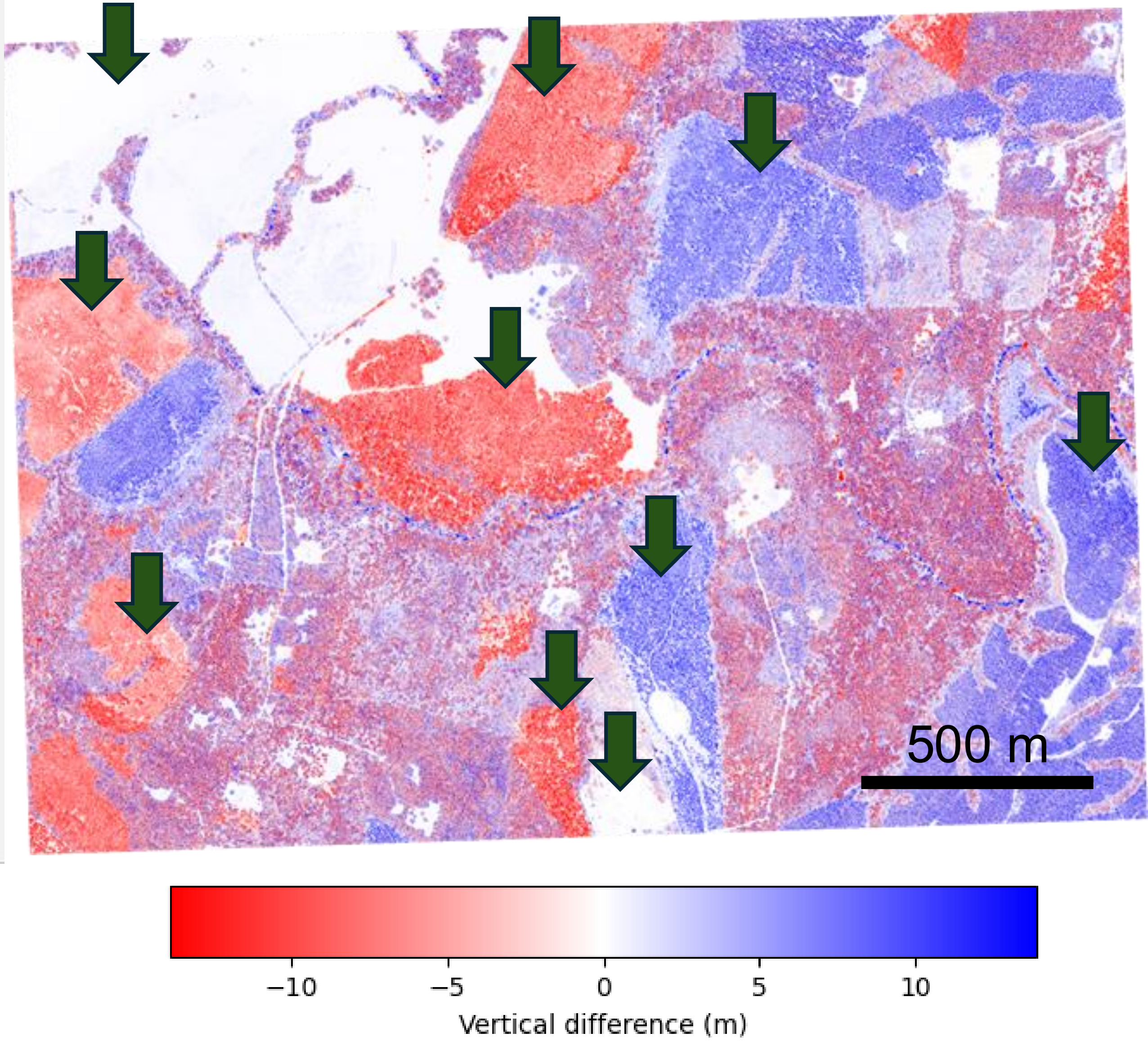




Error in Vertical Differencing

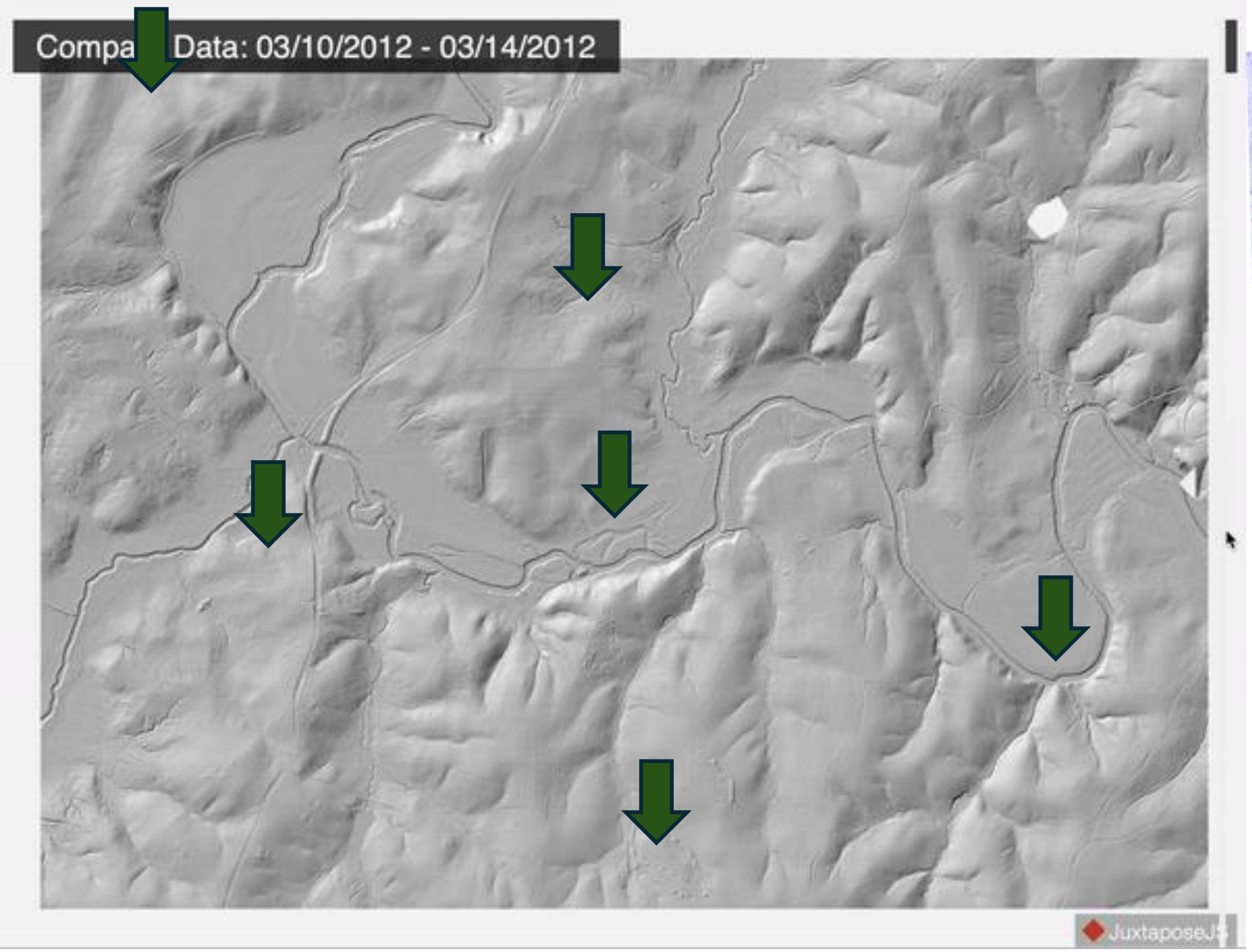
DSM differencing

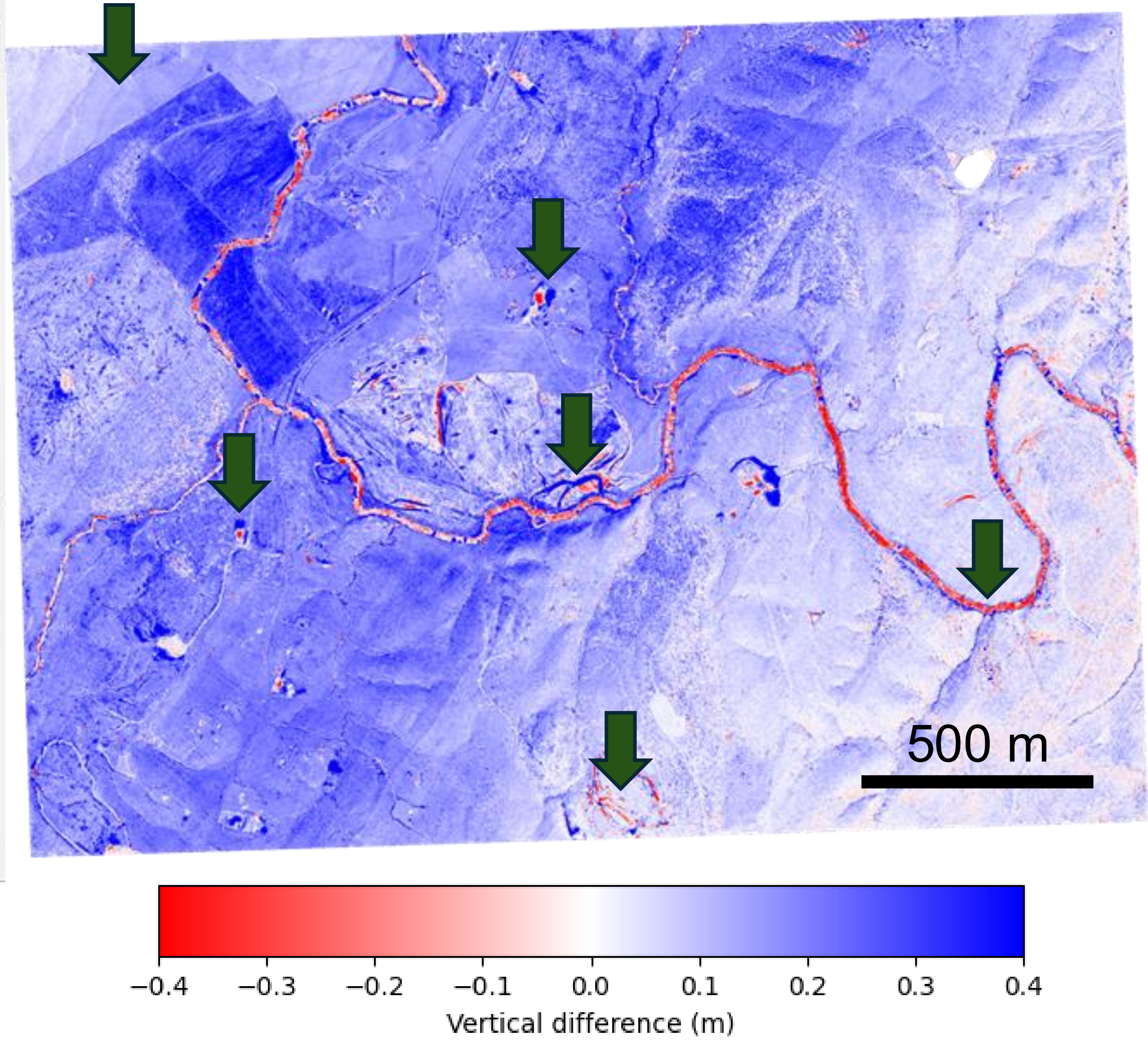


Roundabout Creek, VA
2012 - 2019

Error in Vertical Differencing

DTM differencing





Roundabout Creek, VA
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The Problem: Error in Differencing

Observed change = True change + Error

Older (legacy) datasets typically have more error

Error sources are often similar in magnitude to true change

We need tools to separate signal from noise

Error vs. Uncertainty in Differencing

Error

Signed difference between the measured vertical change and the actual vertical change

$$\text{Error} = \Delta_{z_{measured}} - \Delta_{z_{actual}}$$

E.g., –20 cm

Uncertainty

Spread in possible values of vertical change over a specific area given the presence of error

$$\text{Uncertainty} = \sqrt{Var\ (\overline{Error})}$$

E.g., $\pm$ 4 cm

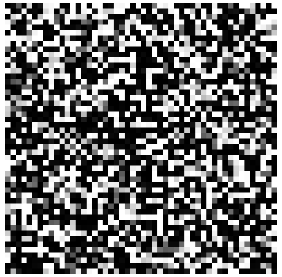
Error in Vertical Differencing

Error is often spatially structured and scale-dependent, not just random noise

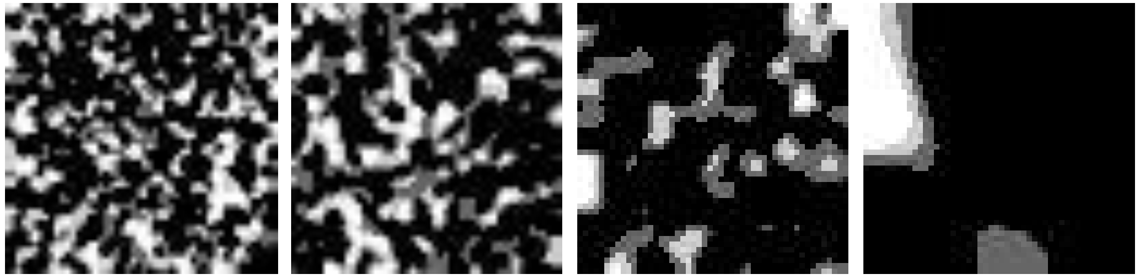
Random

Systematic

Spatially uncorrelated



Spatially correlated



Increasing correlation range length

Sources of Error by Spatial Scale

Long-range (km+):

Calibration bias, incorrect vertical CRS metadata (geoid errors, ellipsoidal vs. orthometric confusion)

Mid-range (100s of m to km):

Horizontal alignment errors (topographically correlated), flight line striping

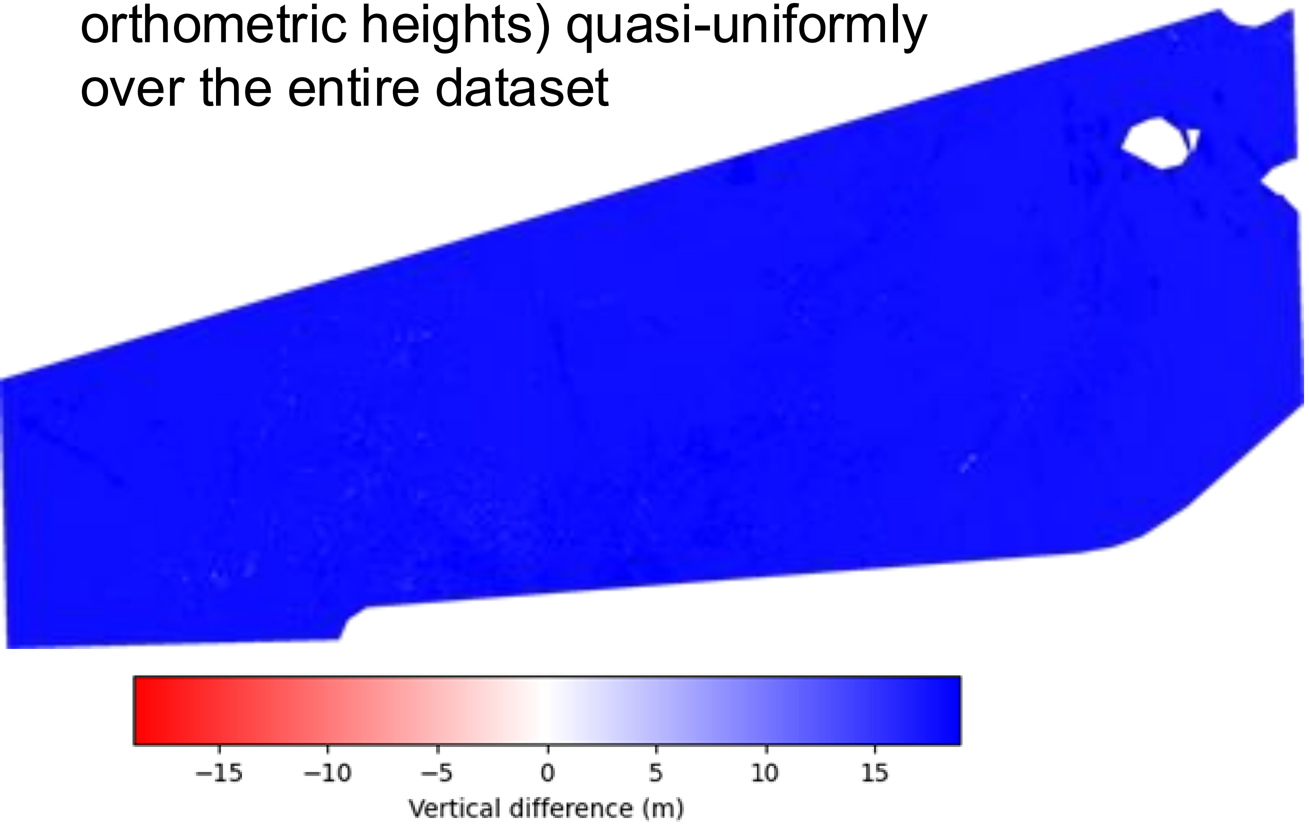
Short-range (m to 10s of m):

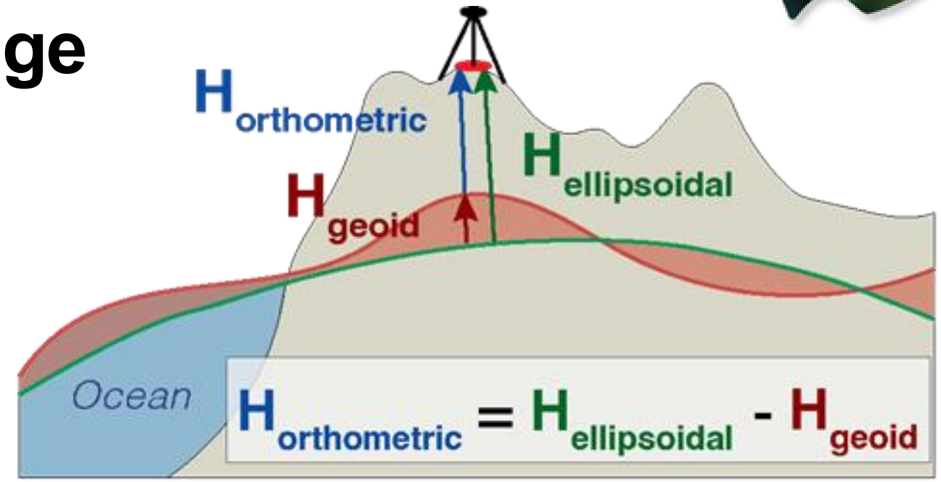
Random noise, point misclassification, geometric distortion on steep slopes

Sources of Error by Spatial Scale

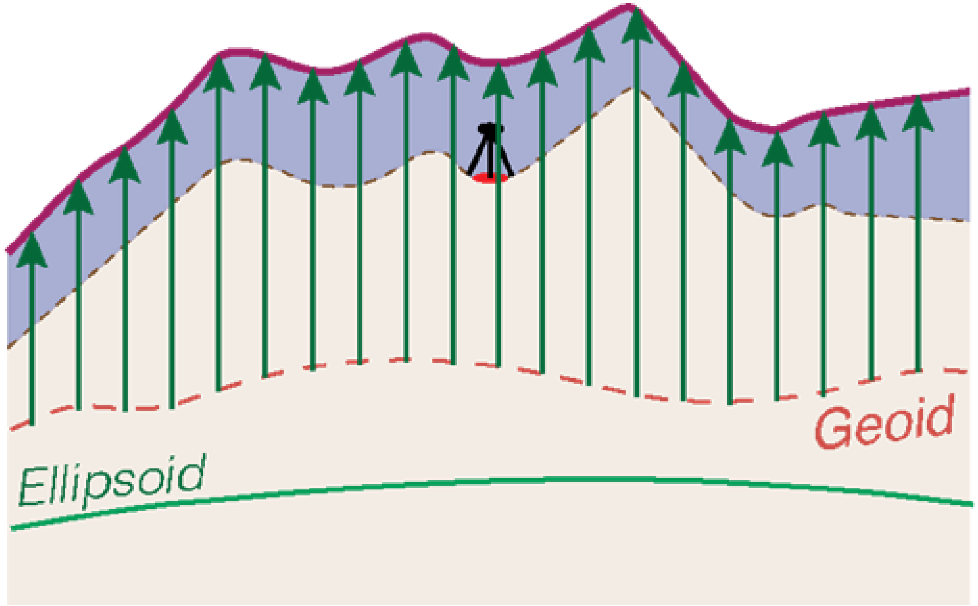
Long-range (km+): **Systematic long-range error due to metadata errors**

Apparent vertical changes from 10-20 cm (incorrect geoid) to tens of meters (incorrect reporting of ellipsoidal and orthometric heights) quasi-uniformly over the entire dataset



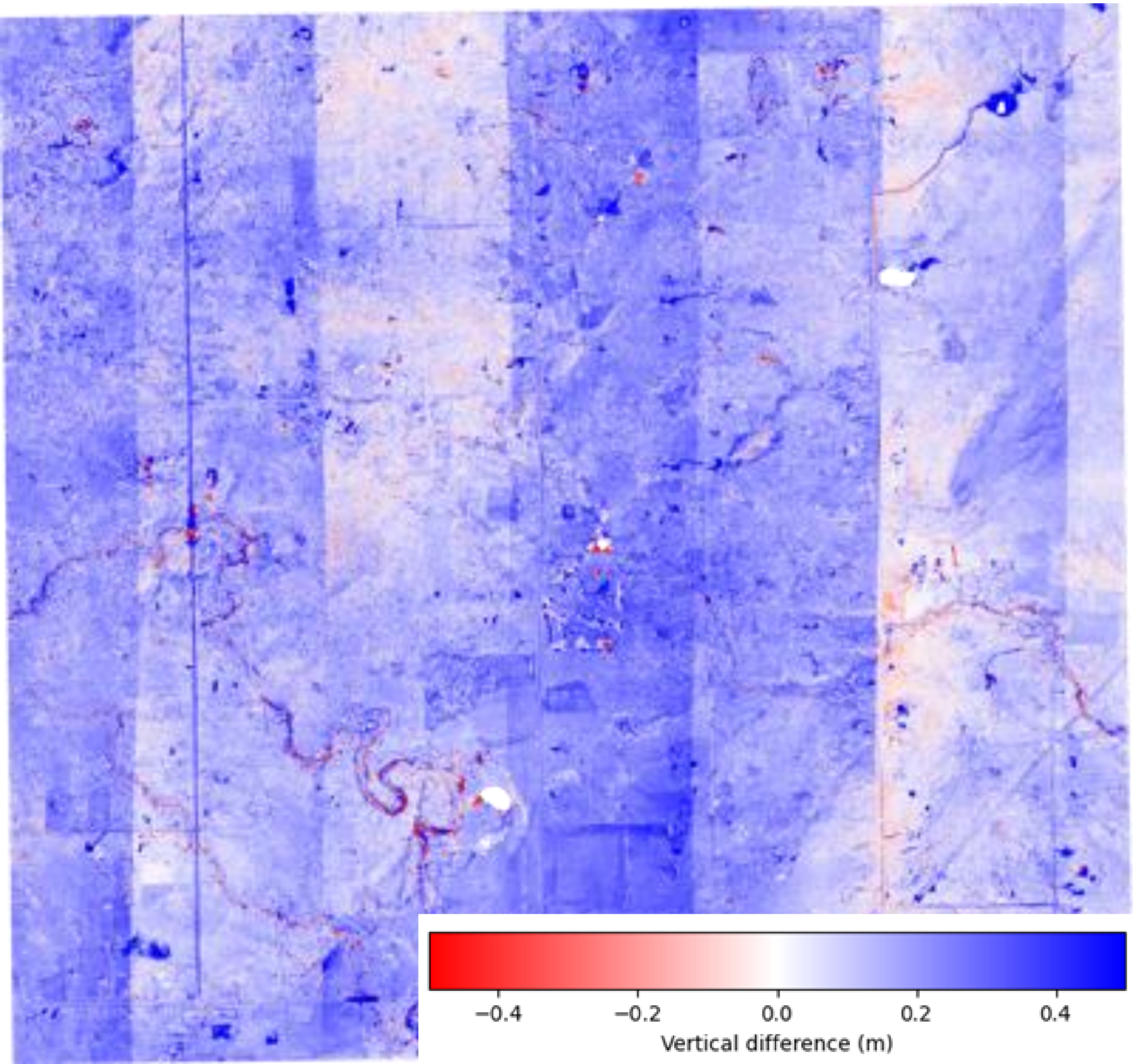


Modified from SERC



Sources of Error by Spatial Scale

Long-range (km+): **Flight line error**



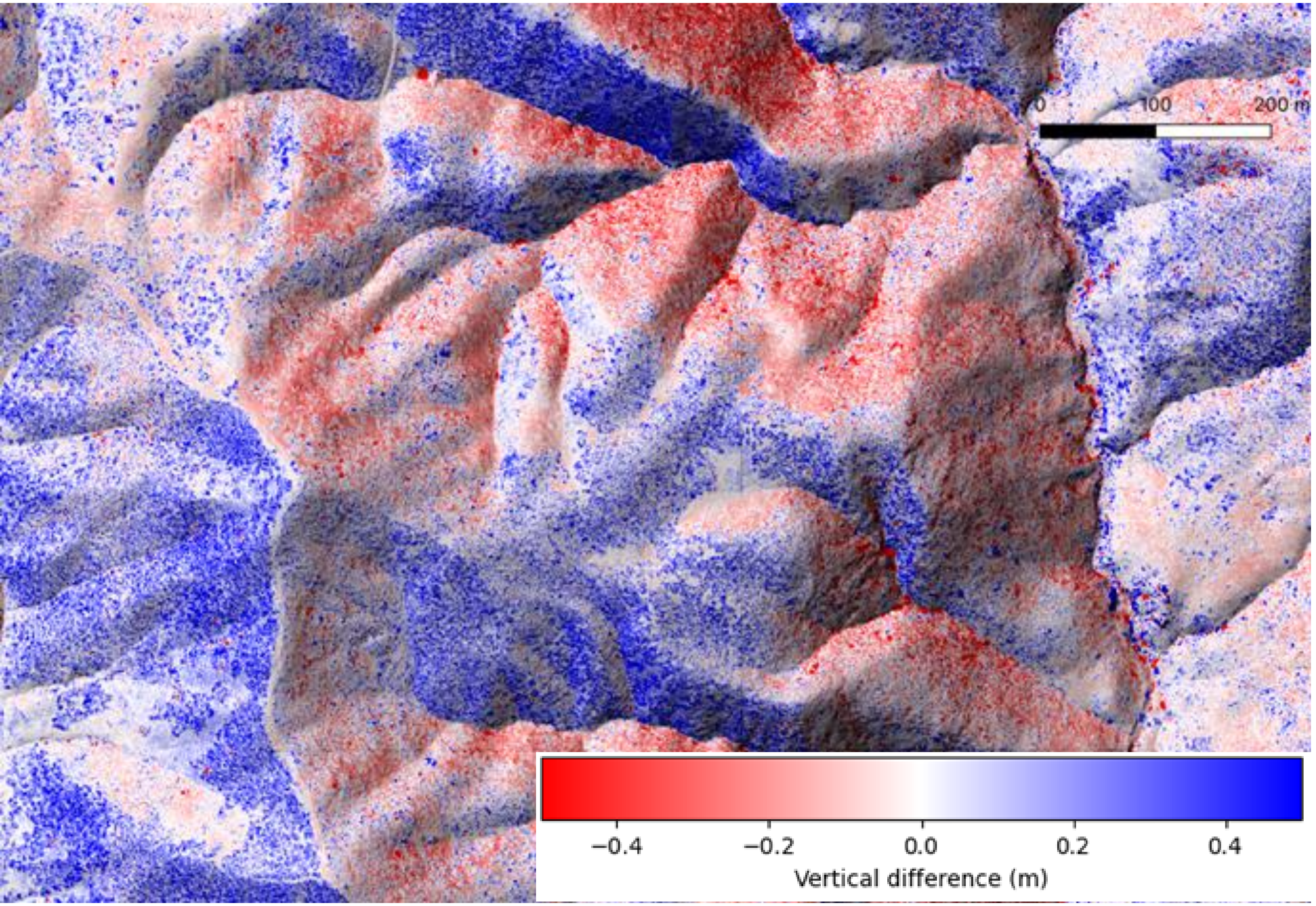
Georeferencing errors of the flight lines make linear, swath-to-swath artifacts

Up to 10's of centimeters of apparent vertical change

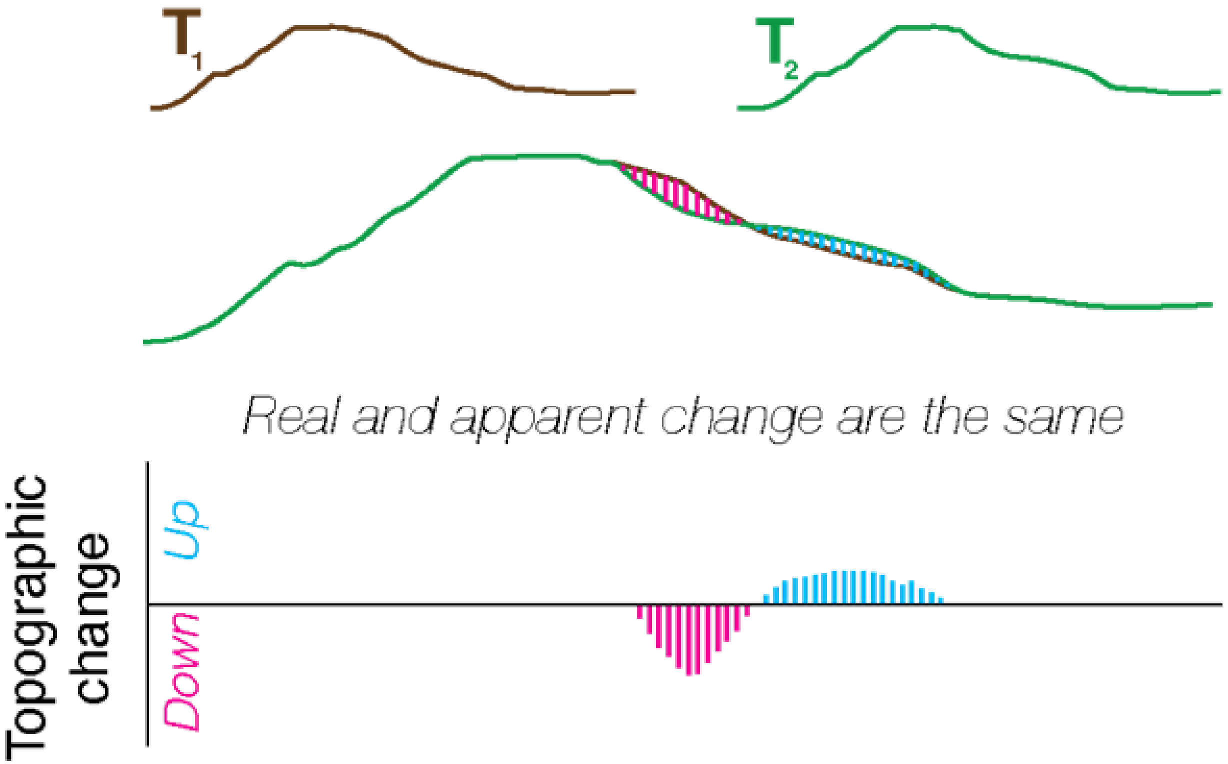
Interferes with linear patterns (e.g., roads, faults, canals) of surface change.

Sources of Error by Spatial Scale

Mid-range (10s to 100s of m): Apparent vertical displacement from incorrectly aligned datasets



If datasets are perfectly co-registered:



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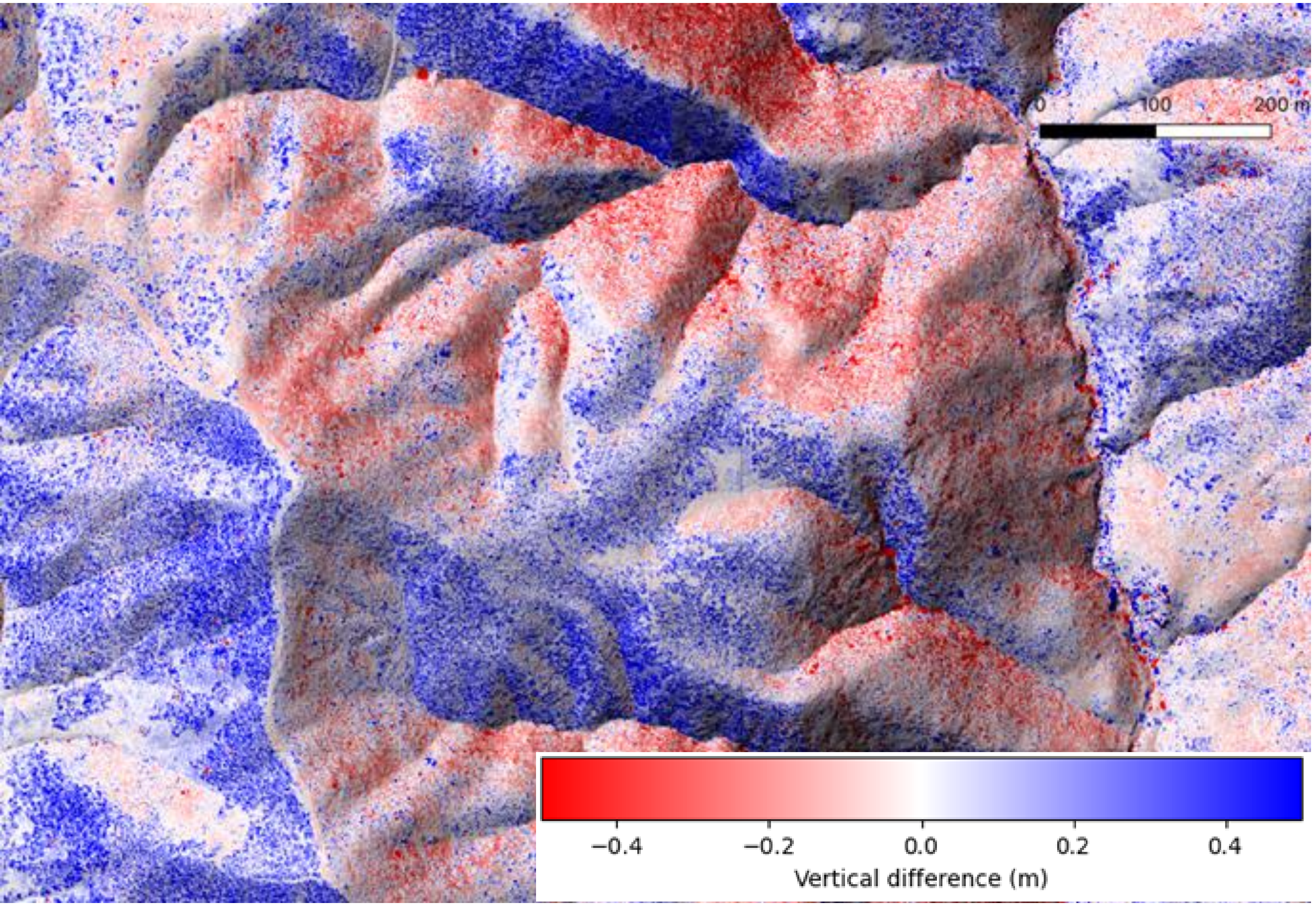
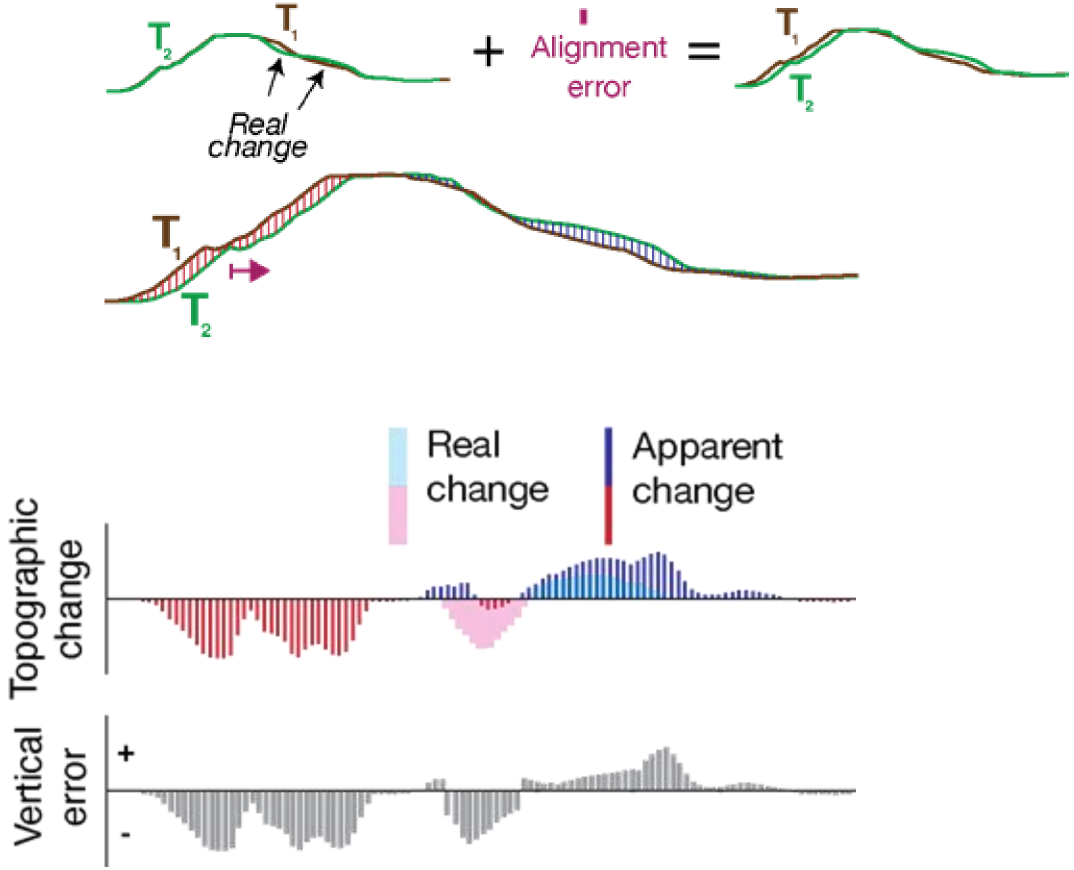
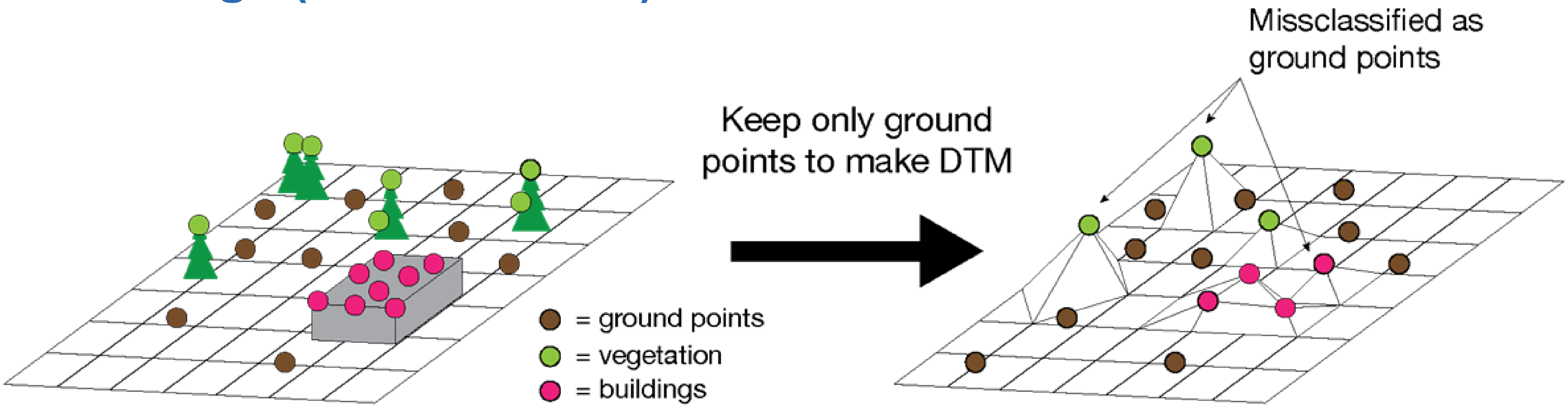


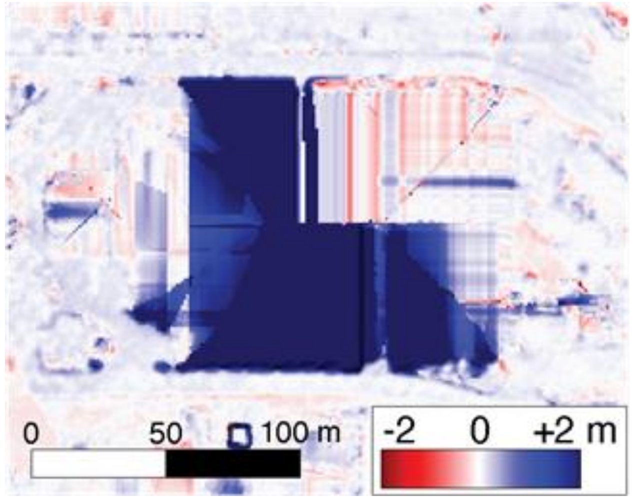
If datasets are horizontally displaced:



Sources of Error by Spatial Scale

Short-range (m to 10s of m): **Misclassification error**

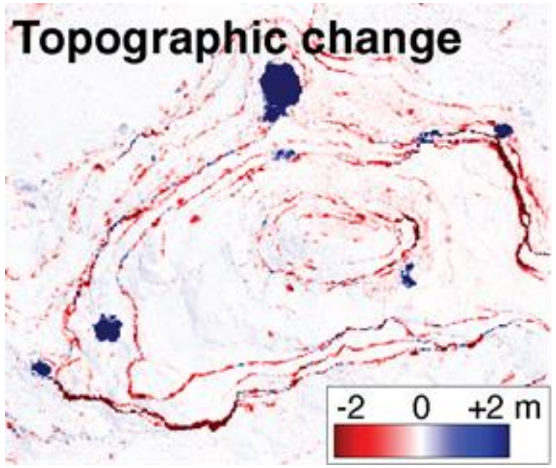


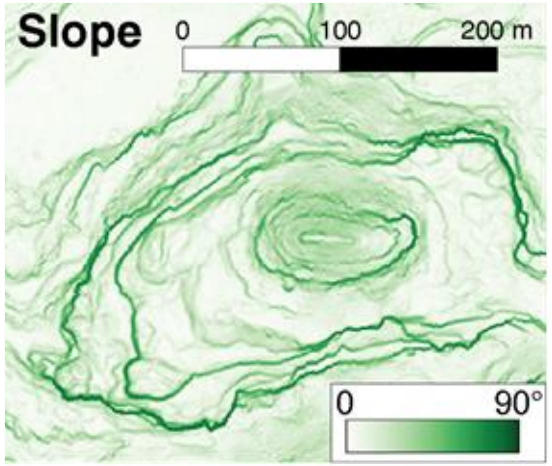


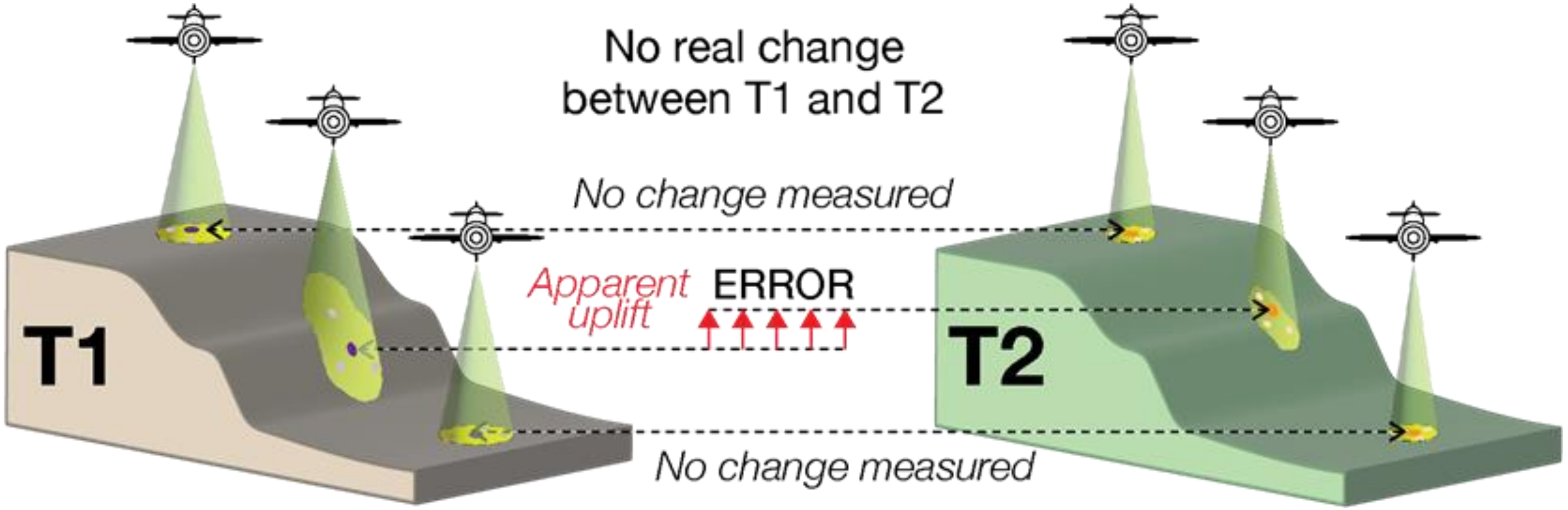
If ground is incorrectly or inconsistently classified, generation of incorrect DTM

Sources of Error by Spatial Scale

Short-range (m to 10s of m): Geometric distortion in high-slope areas

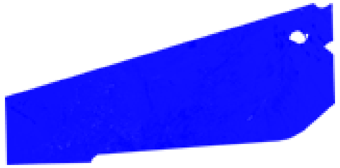




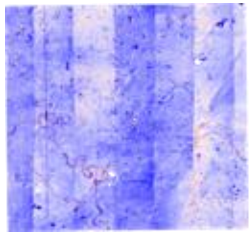


Lidar sensor oblique to the ground surface leads to greater artificial horizontal offsets, especially over steep slopes.

Uncertainty in Vertical Differencing



Very long range



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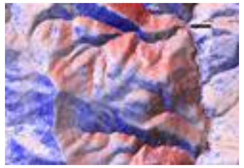

Mid range

Short range

Very short range

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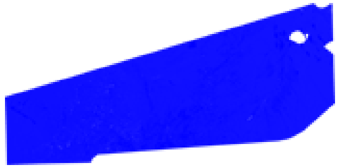
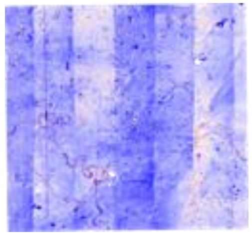


Very long range

Vertical bias

– 0.03 m



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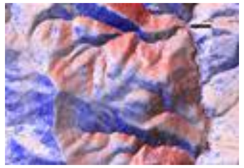

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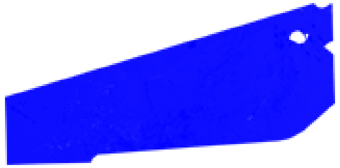

Very long range → **Vertical bias**

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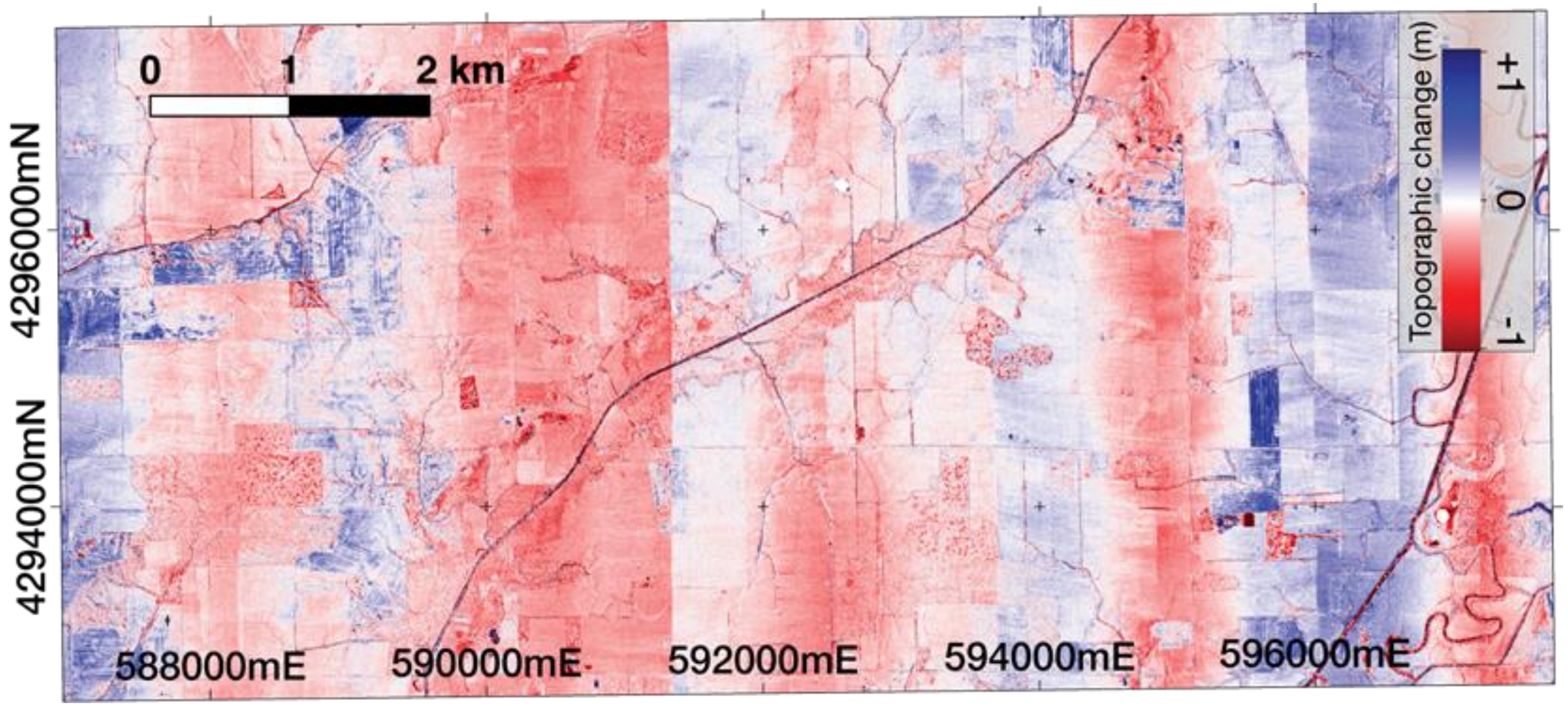

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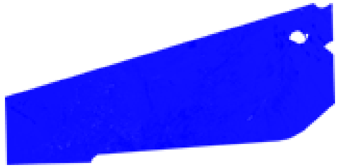

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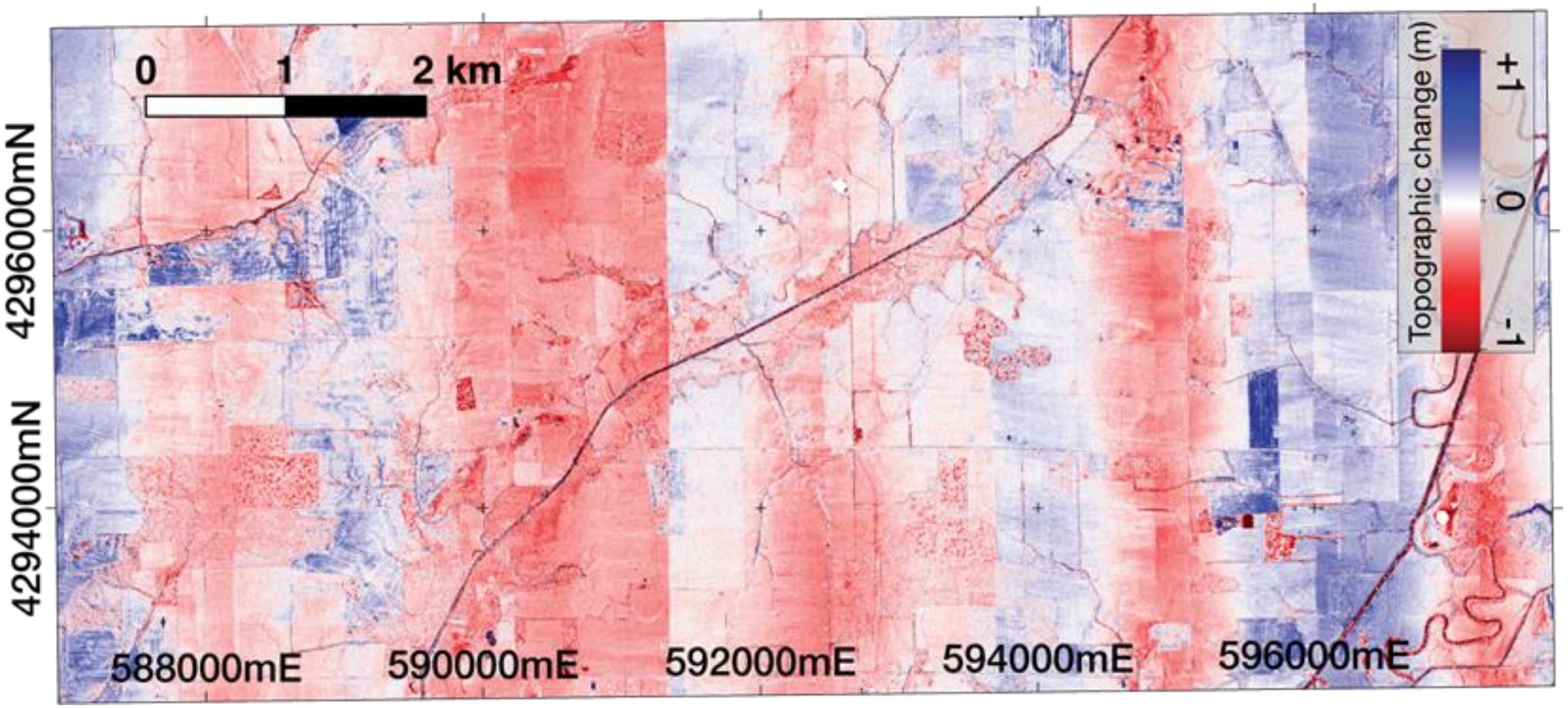

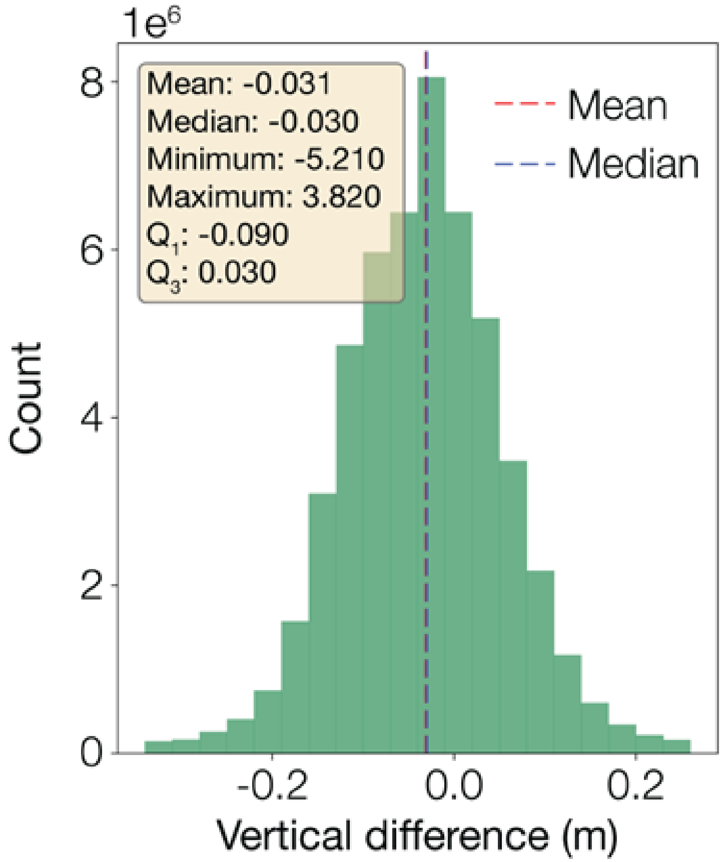

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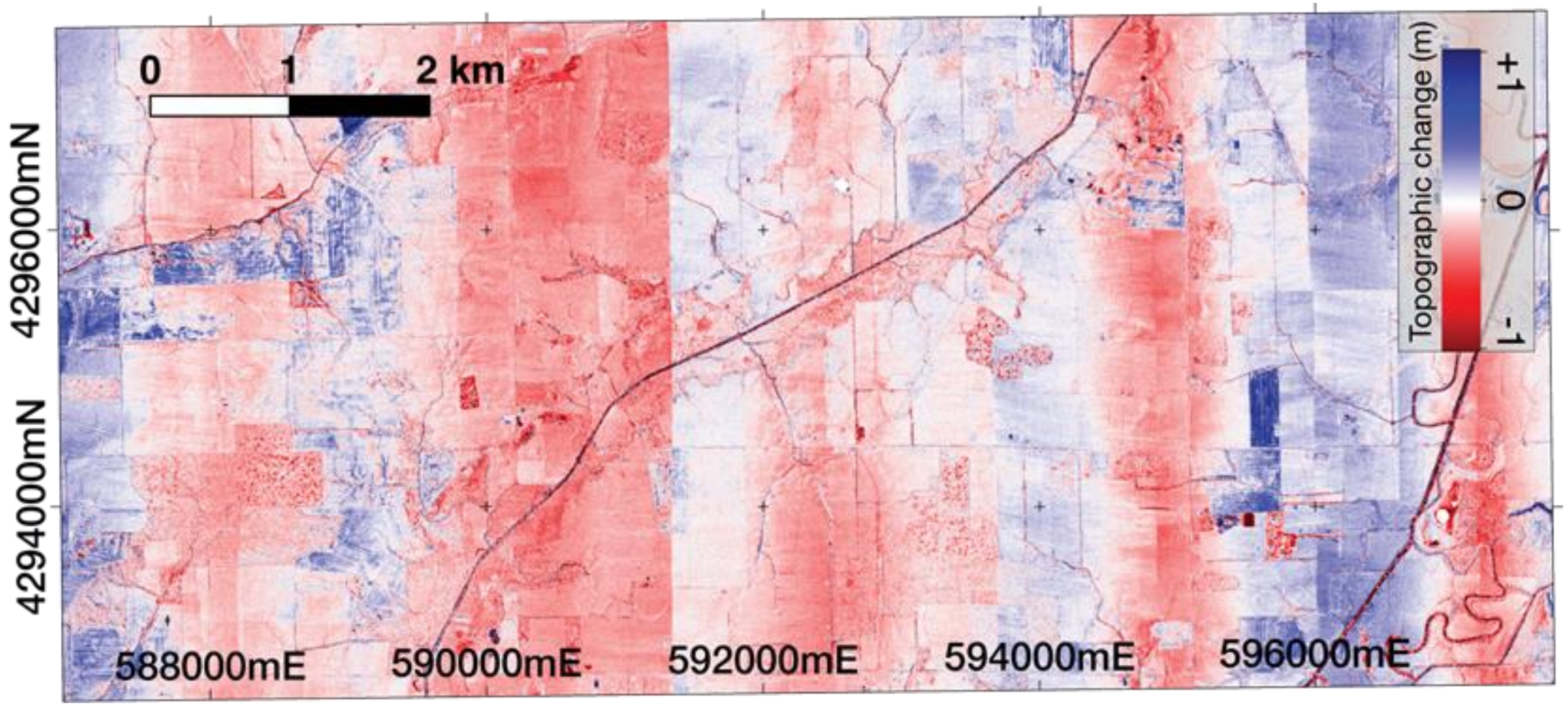

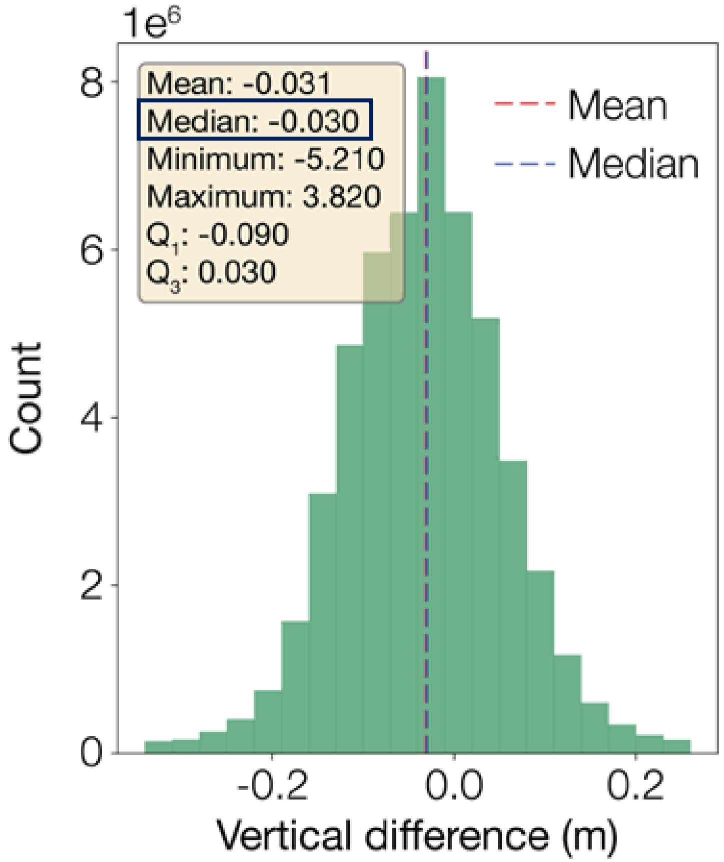

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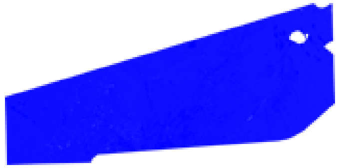

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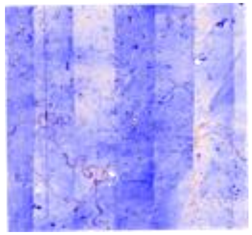

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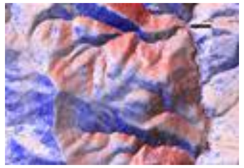
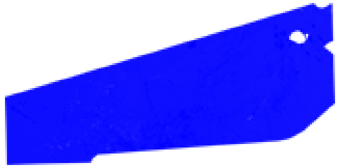


Mid range

Short range

Mean correlated uncertainty values over several length scales

Uncertainty in Vertical Differencing



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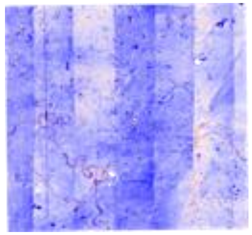

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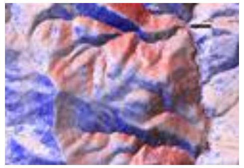

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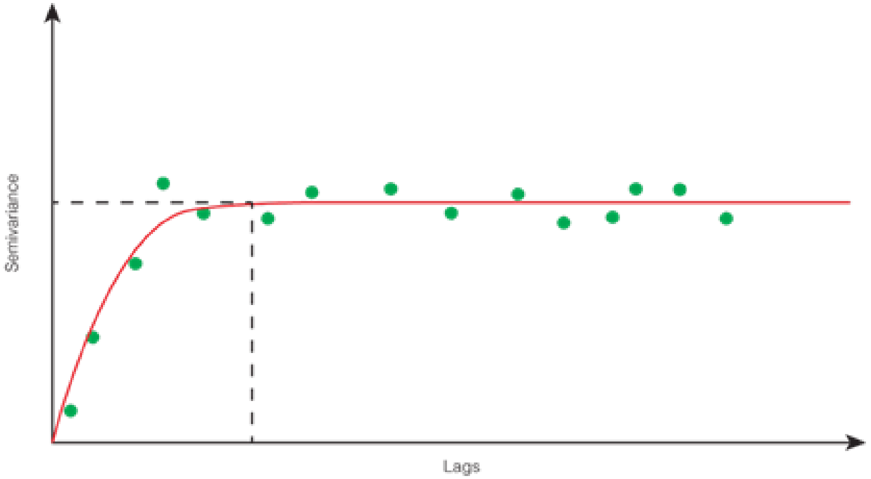
→ **Mean correlated uncertainty values over several length scales**



Variogram analysis

Questions?

- Uncertainty and error in lidar datasets and vertical differencing results
- Sources of error by spatial scale
  - Very long range (km+): e.g., geoid errors, confusion between orthometric and ellipsoidal coordinates
  - Long range (100s of m to km): e.g., flight line error
  - Mid range (10s to 100s of m): e.g., horizontal mis alignment
  - Short range (m to 10s of m): e.g., point misclassification errors, geometric distortion

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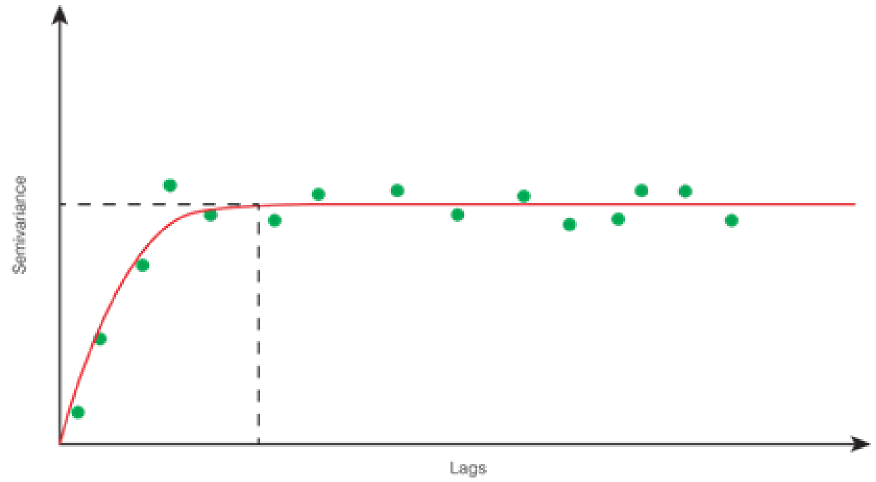
Uncertainty in Vertical Differencing

Variogram analysis



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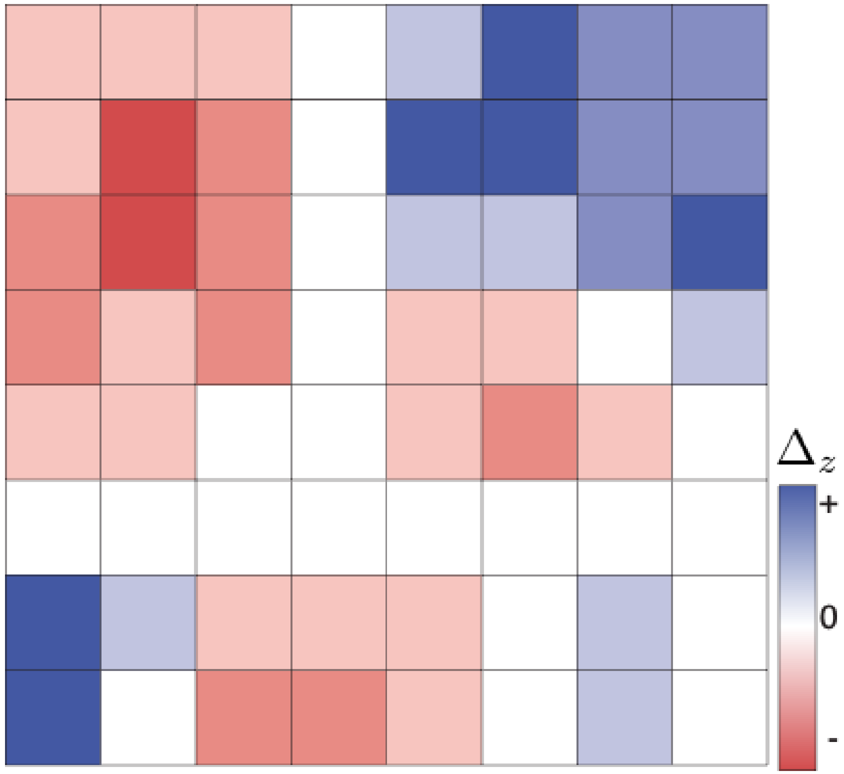

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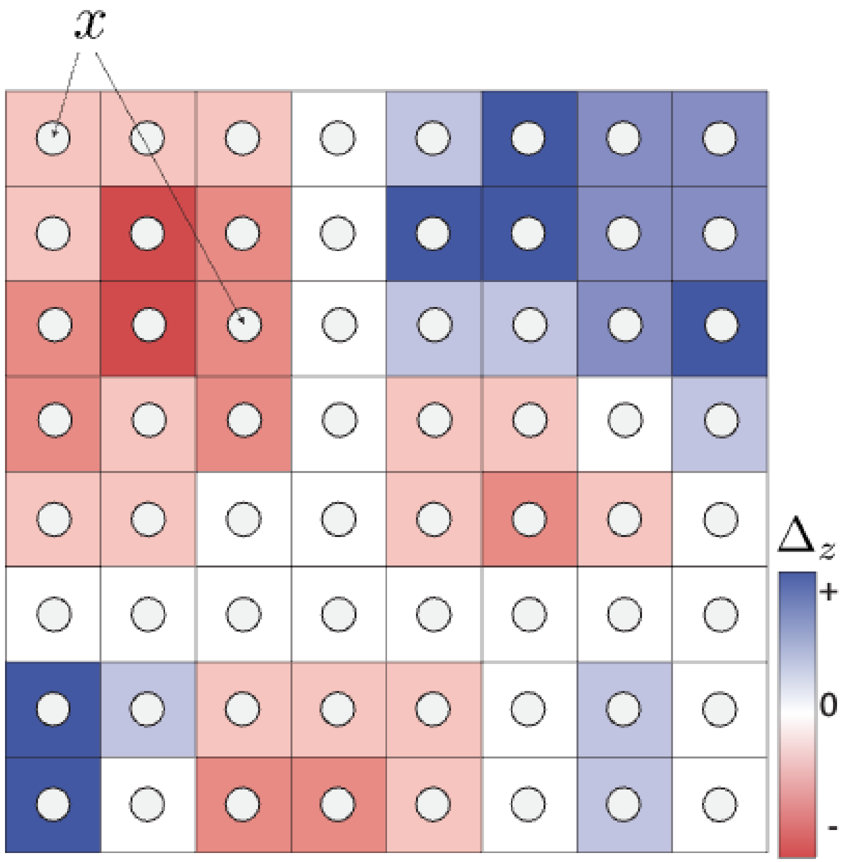

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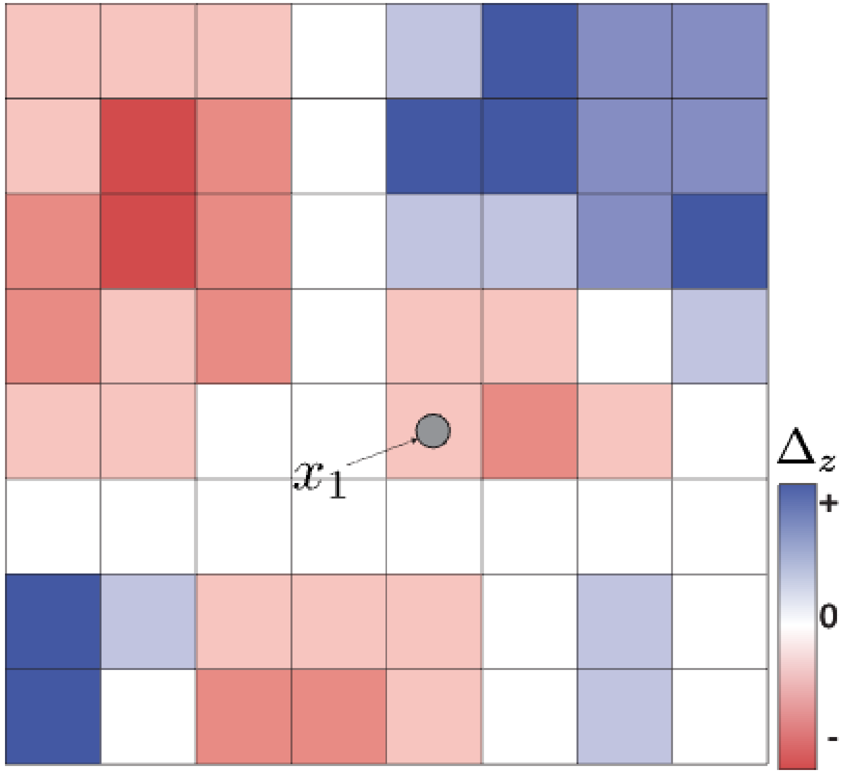

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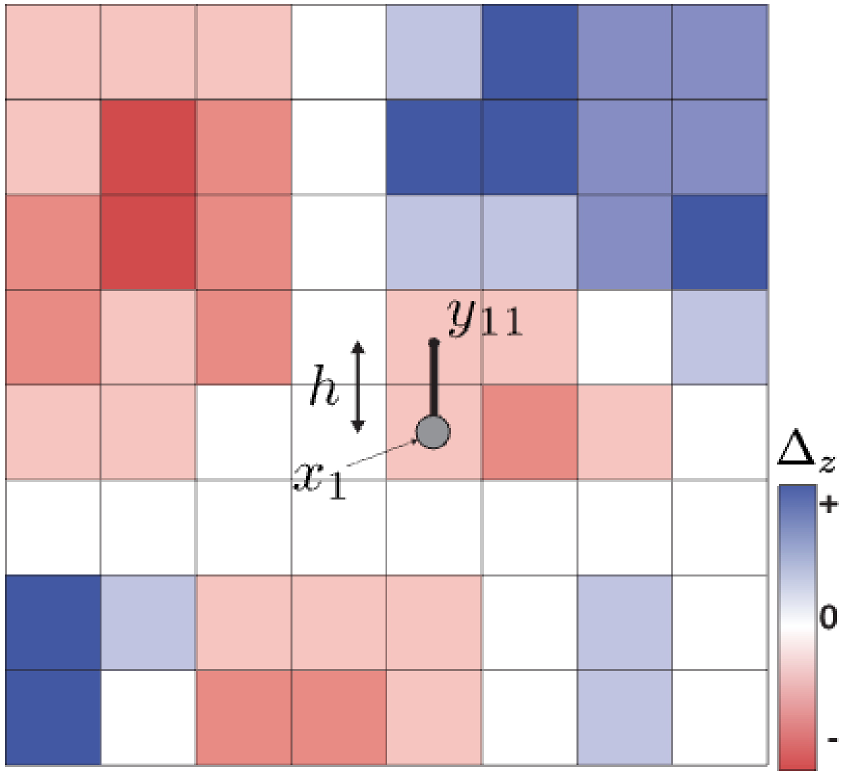

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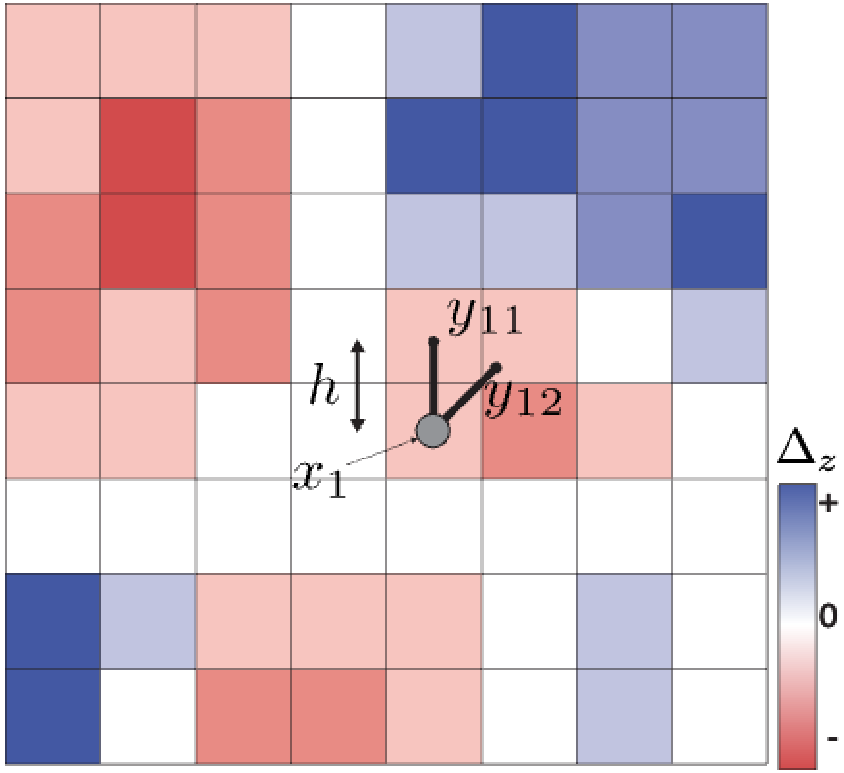

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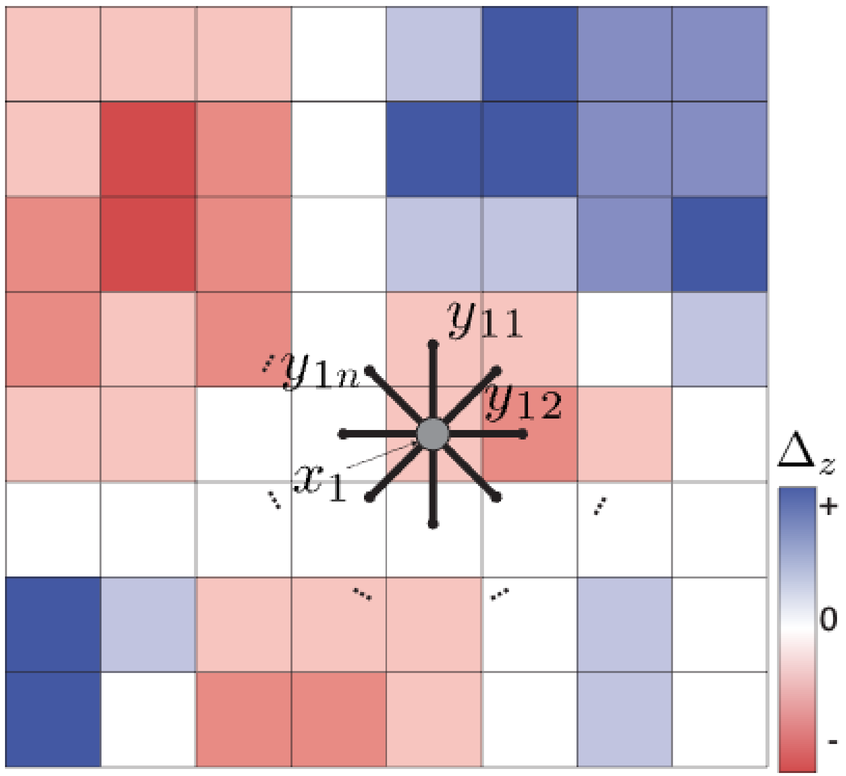

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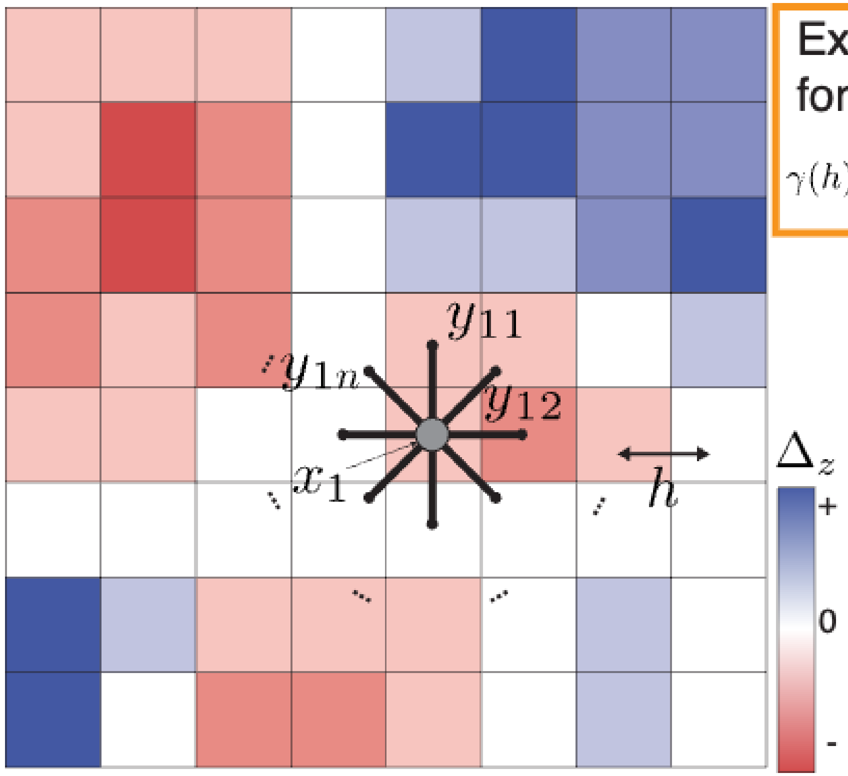
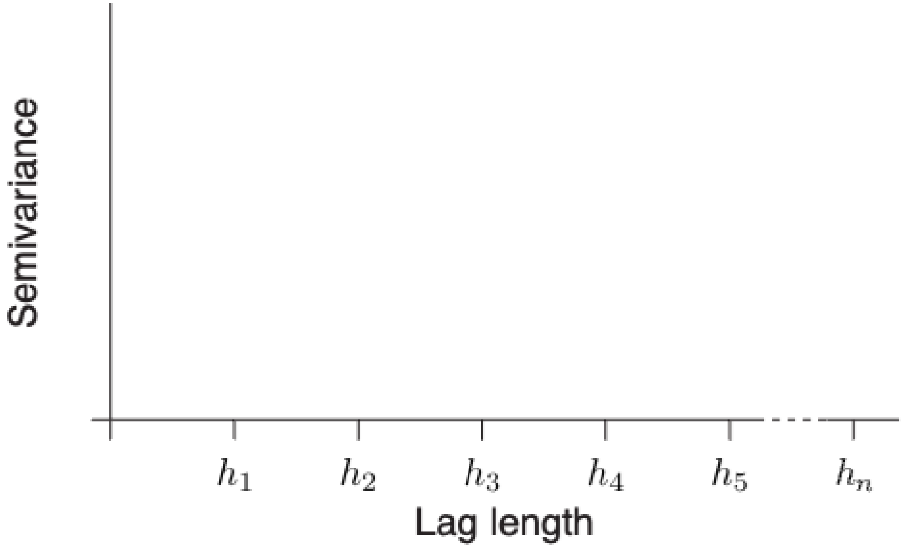


Experimental semivariance calculation for a given lag h, calculated in n directions for m samples :

$$\gamma(h) = \frac{[\Delta_z(x_1) - \Delta_z(y_{h1})]^2 + [\Delta_z(x_1) - \Delta_z(y_{h2})]^2 + \ldots + [\Delta_z(x_2) - \Delta_z(y_{h1})]^2 + \ldots + [\Delta_z(x_m) - \Delta_z(y_{hn})]^2}{2mn}$$



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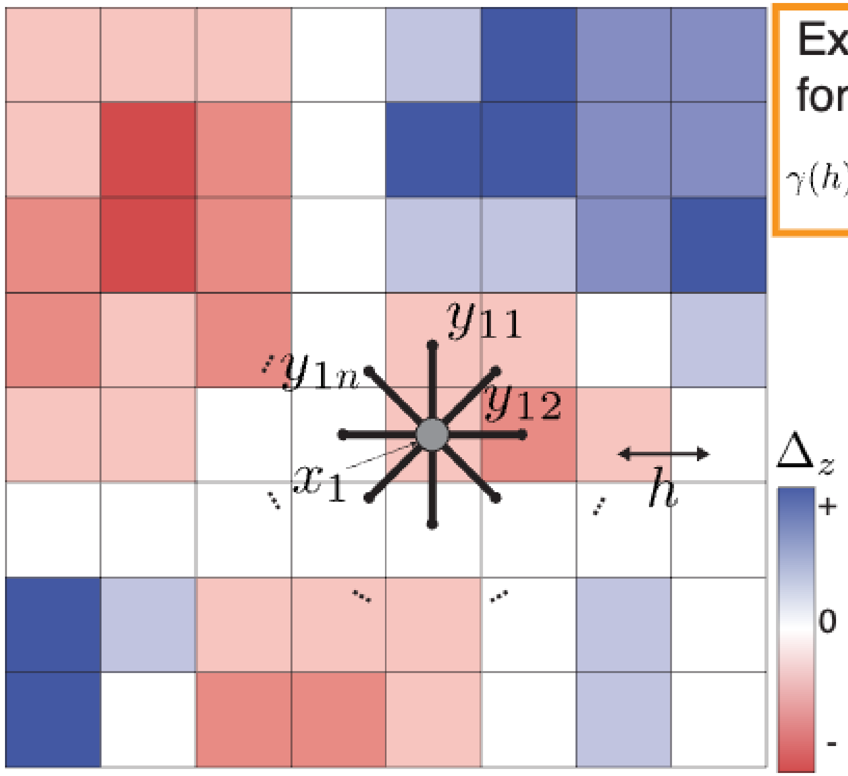

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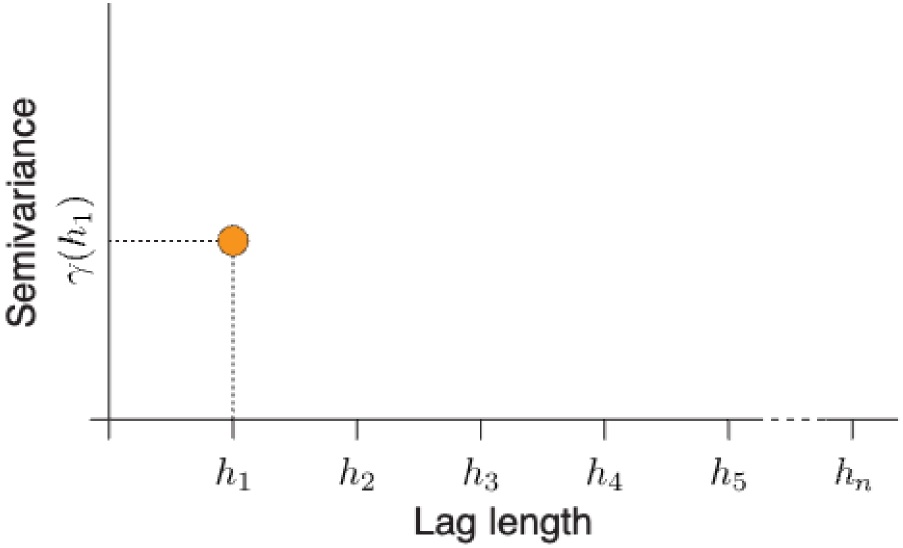

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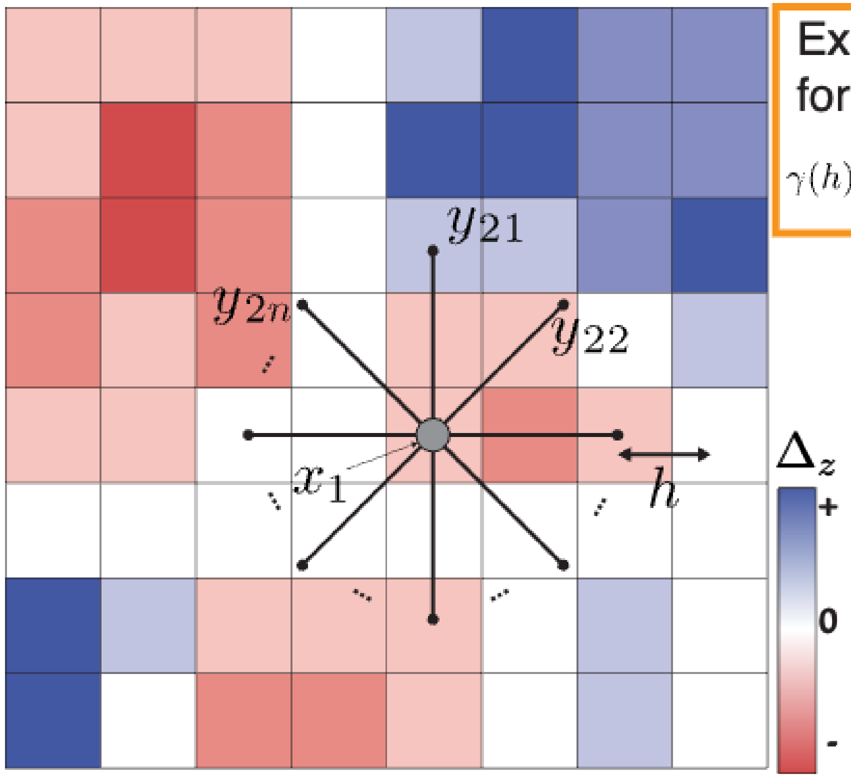

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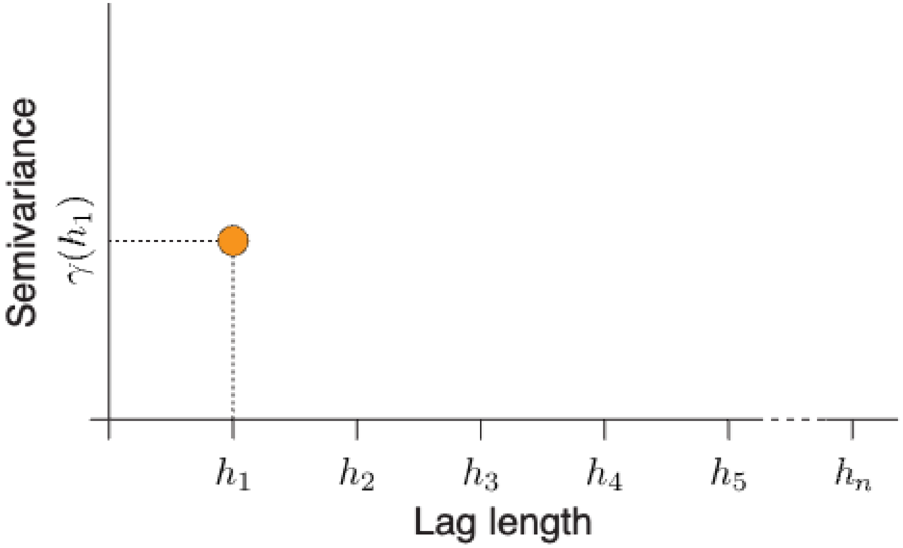

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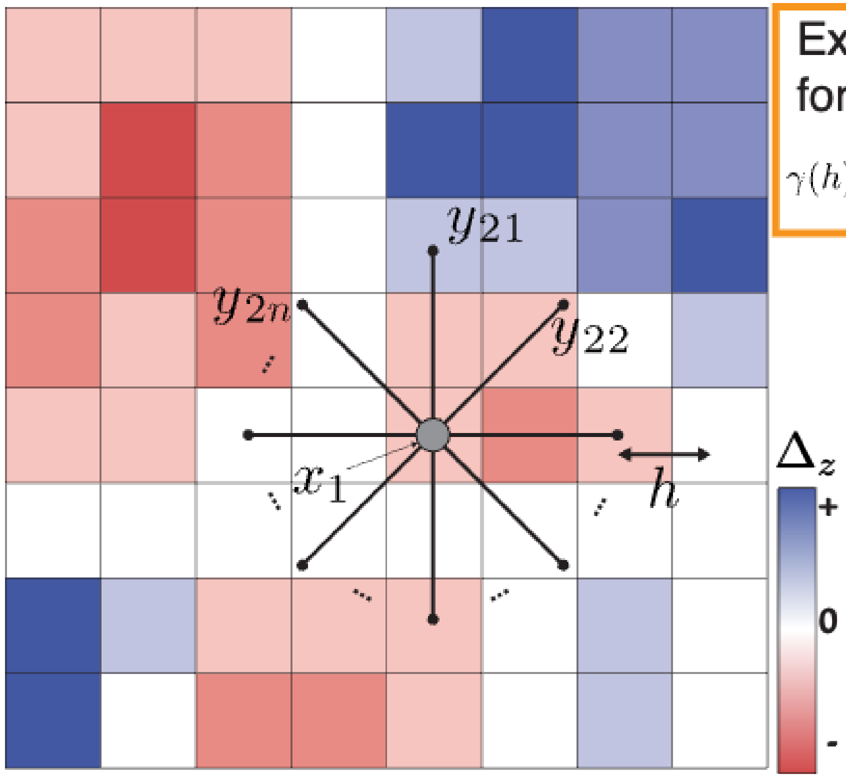

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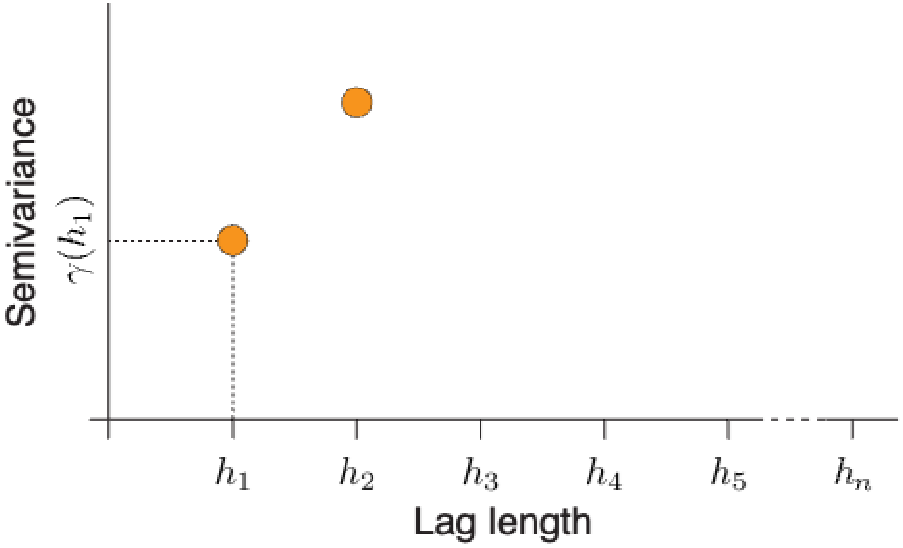
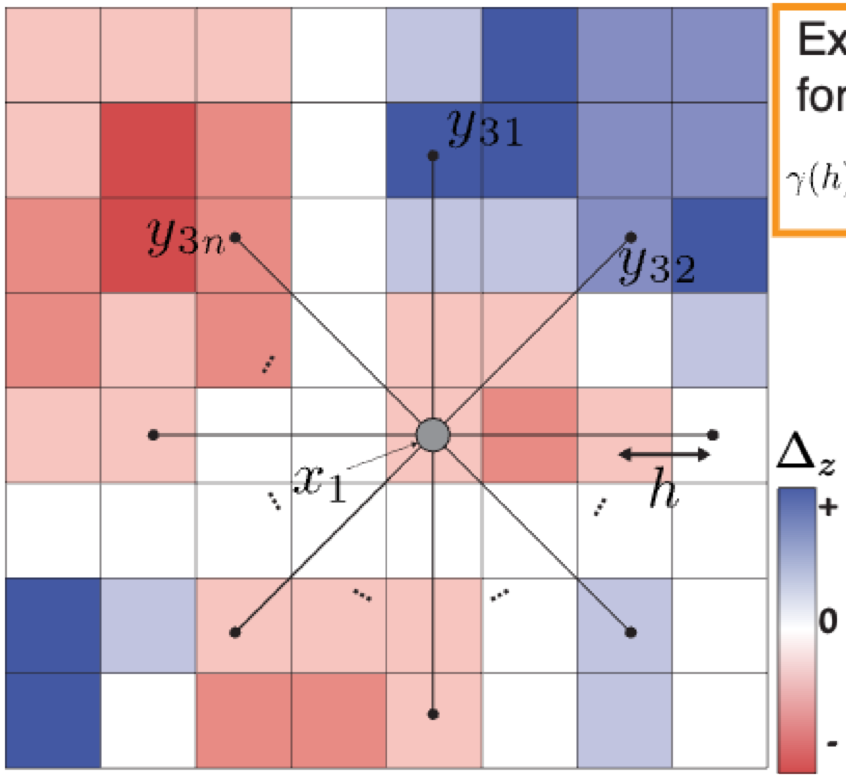
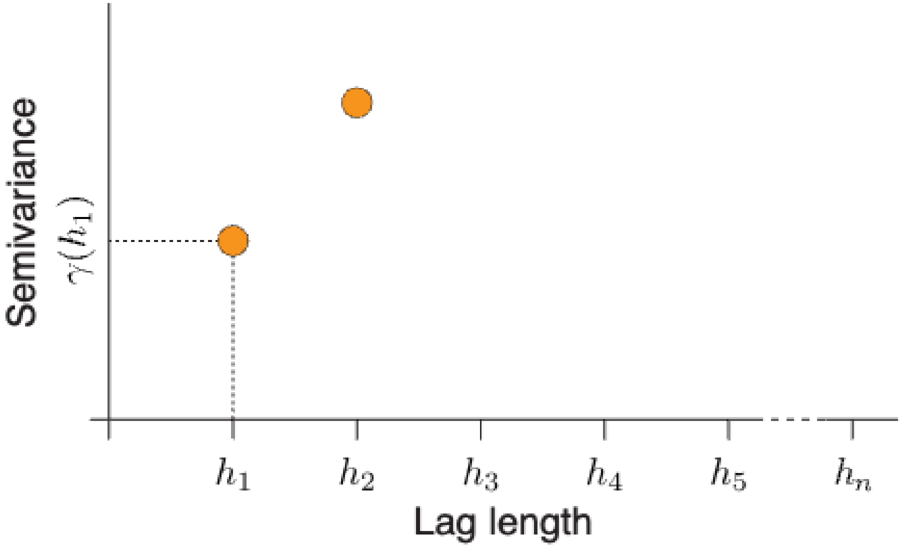


Uncertainty in Vertical Differencing



Experimental semivariance calculation for a given lag h, calculated in n directions for m samples :

$$\gamma(h) = \frac{[\Delta_z(x_1) - \Delta_z(y_{h1})]^2 + [\Delta_z(x_1) - \Delta_z(y_{h2})]^2 + \ldots + [\Delta_z(x_2) - \Delta_z(y_{h1})]^2 + \ldots + [\Delta_z(x_m) - \Delta_z(y_{hn})]^2}{2mn}$$



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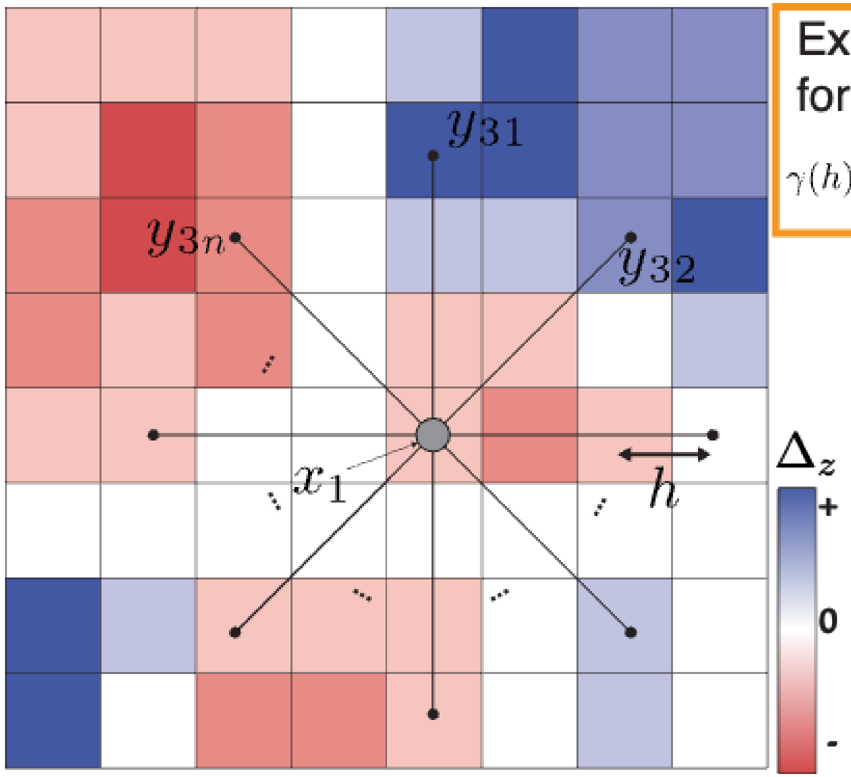

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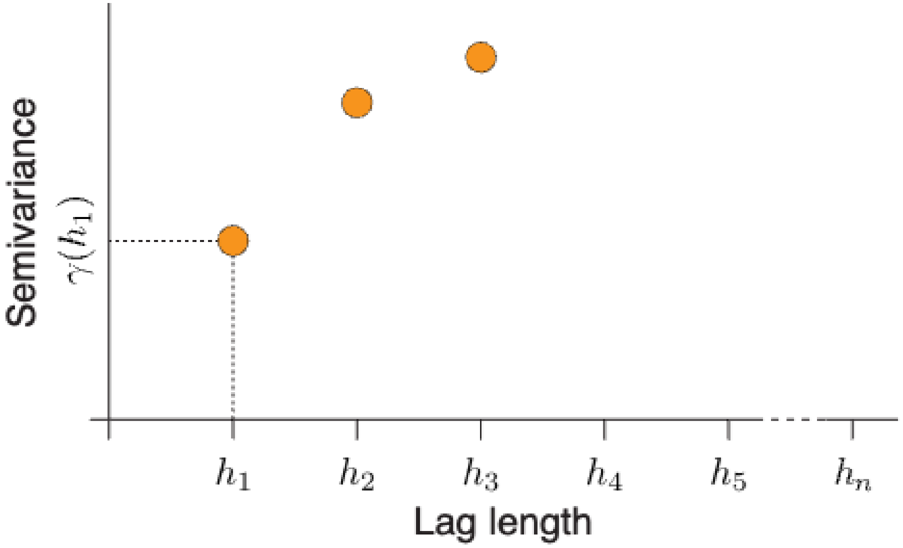
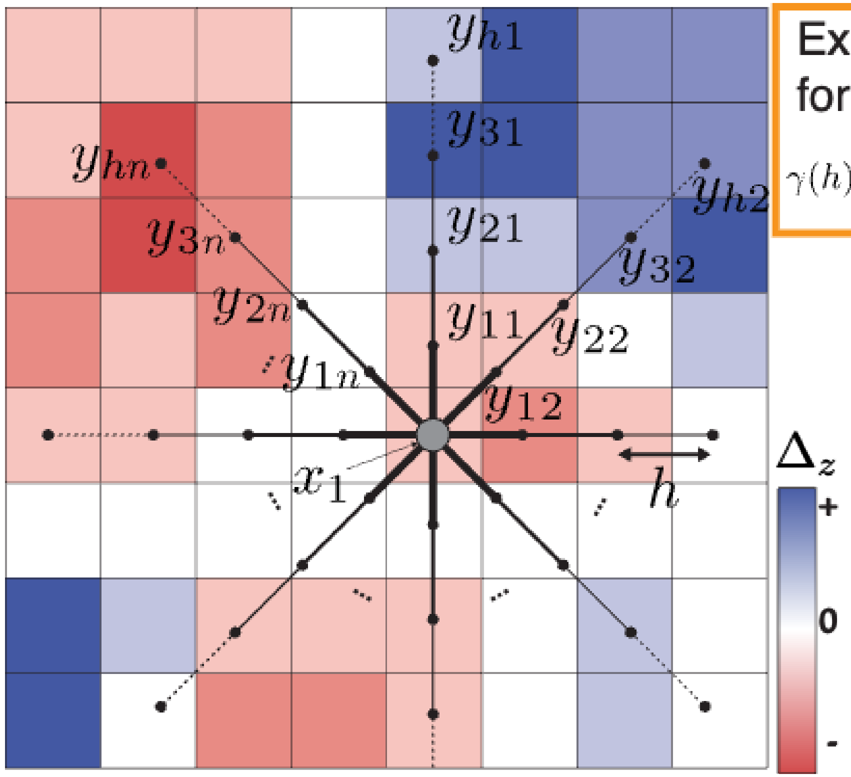


Uncertainty in Vertical Differencing



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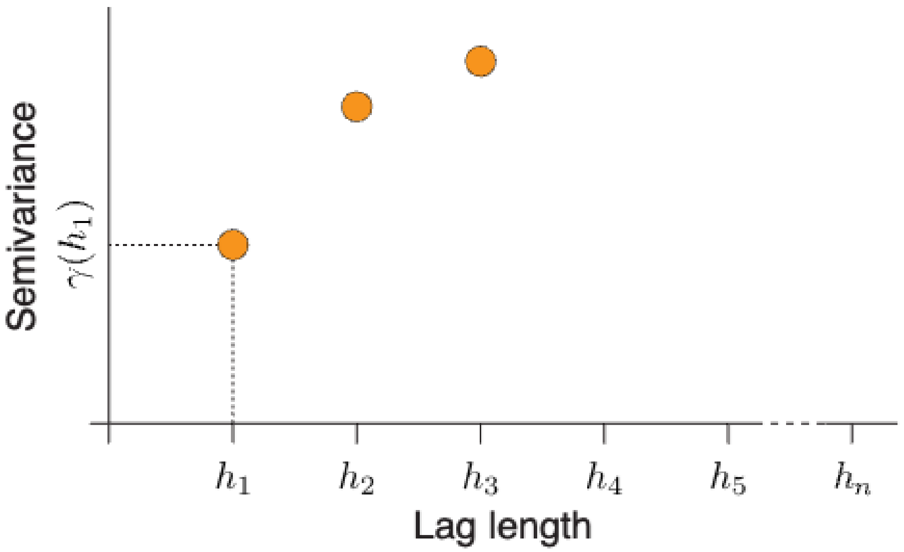
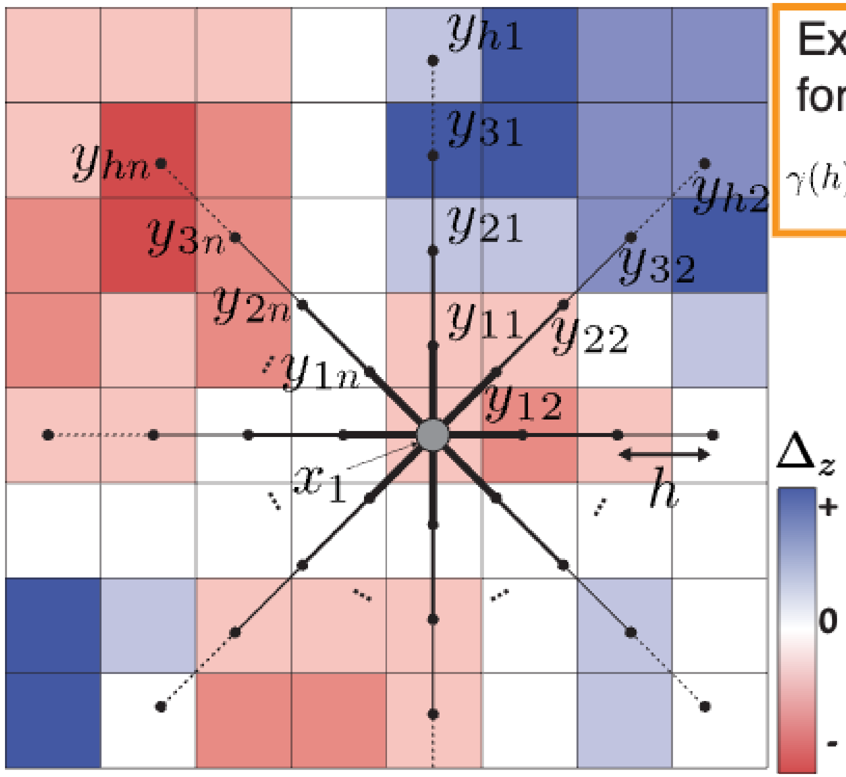


Uncertainty in Vertical Differencing



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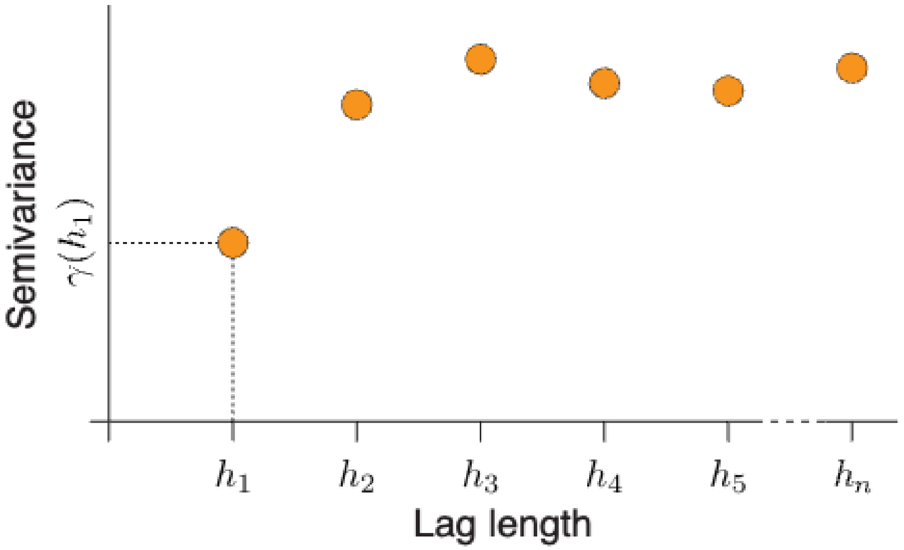

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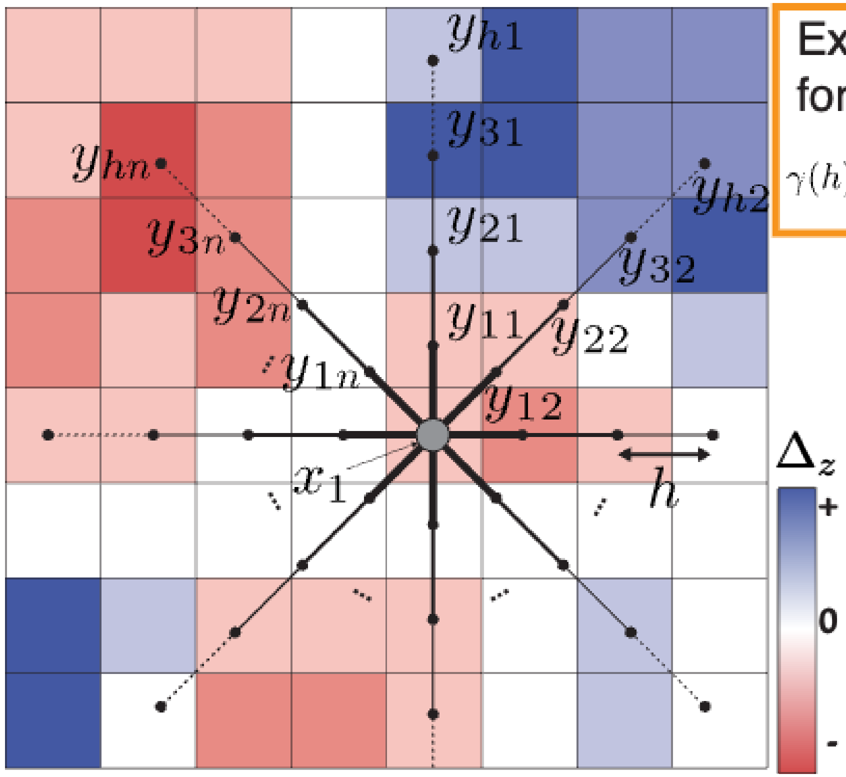

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Semivariance modeled with spherical model of range a and sill c :

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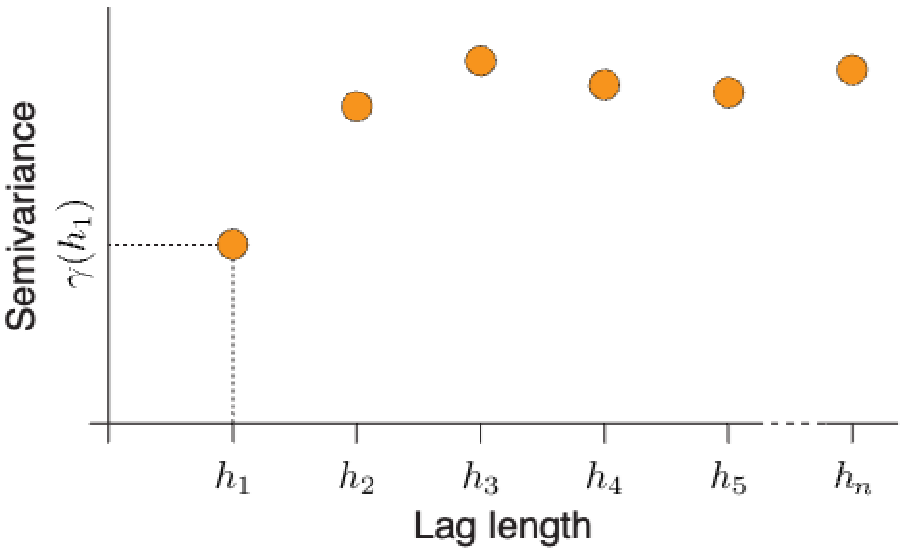
c_0 represents the nugget effect, equal to 0 in this example.



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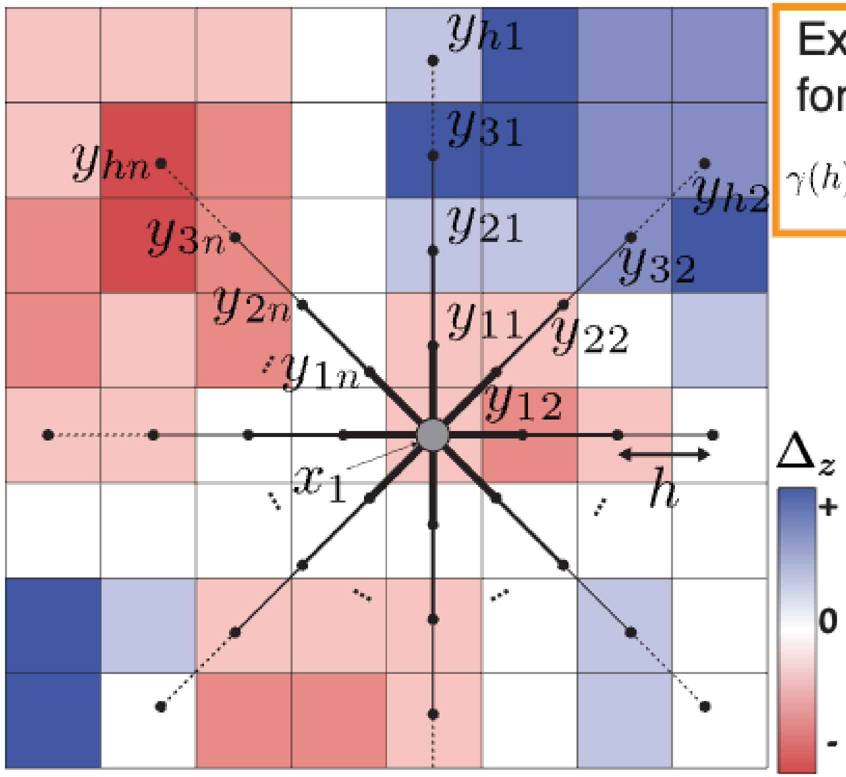

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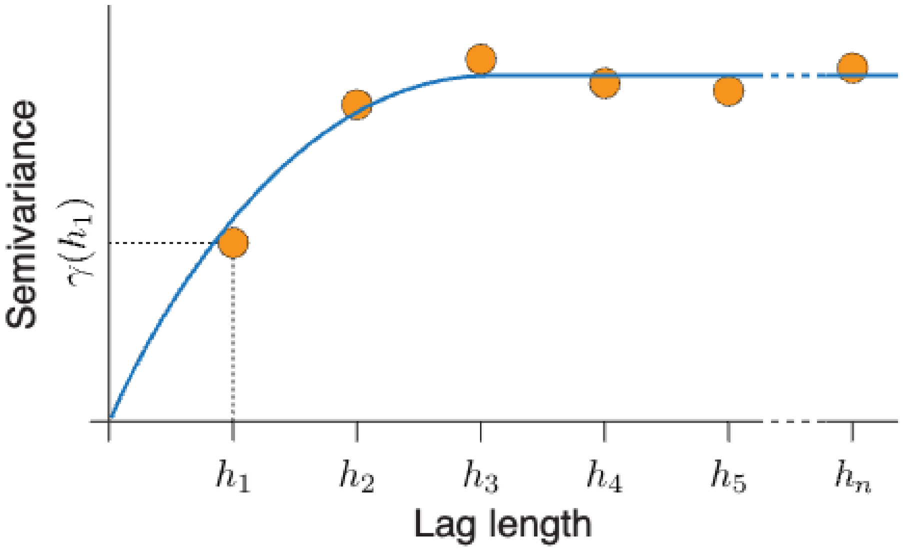

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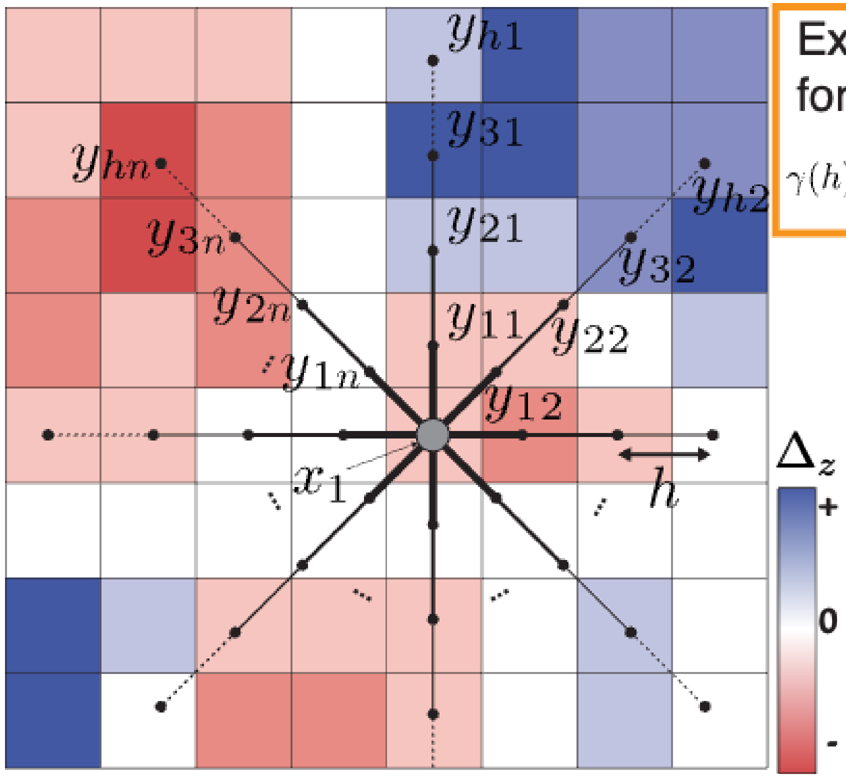

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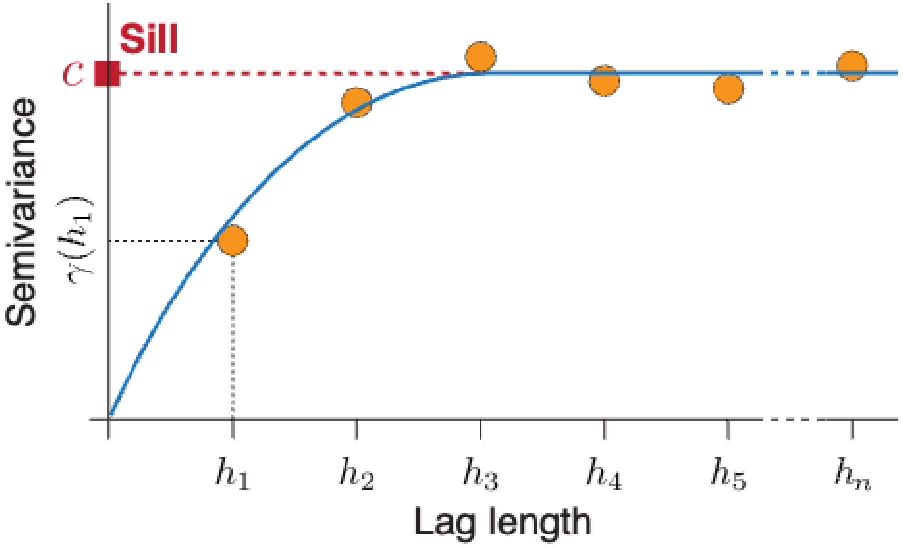
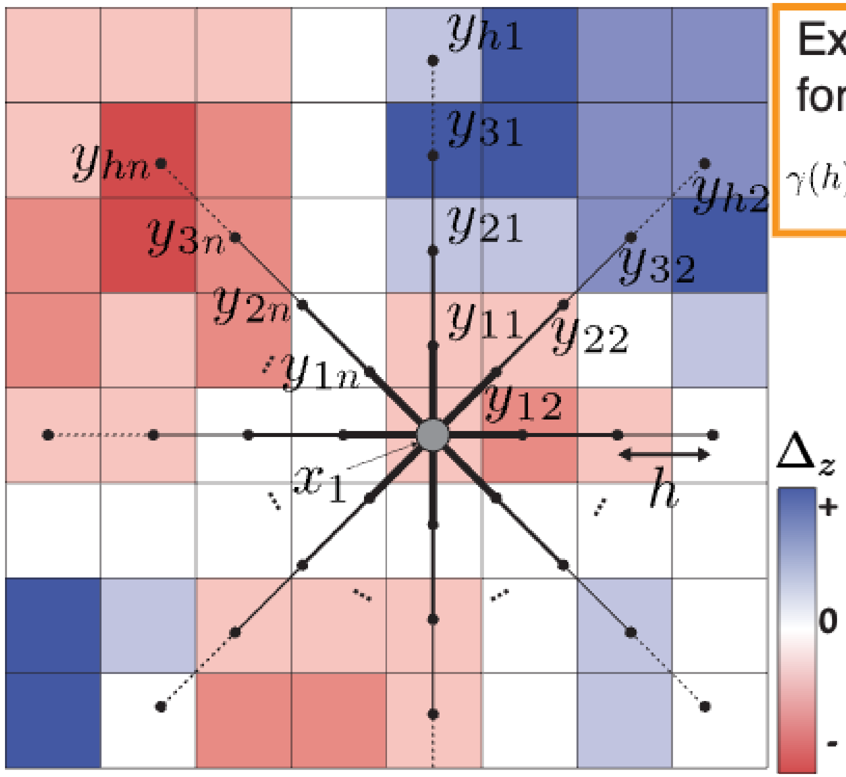


Uncertainty in Vertical Differencing



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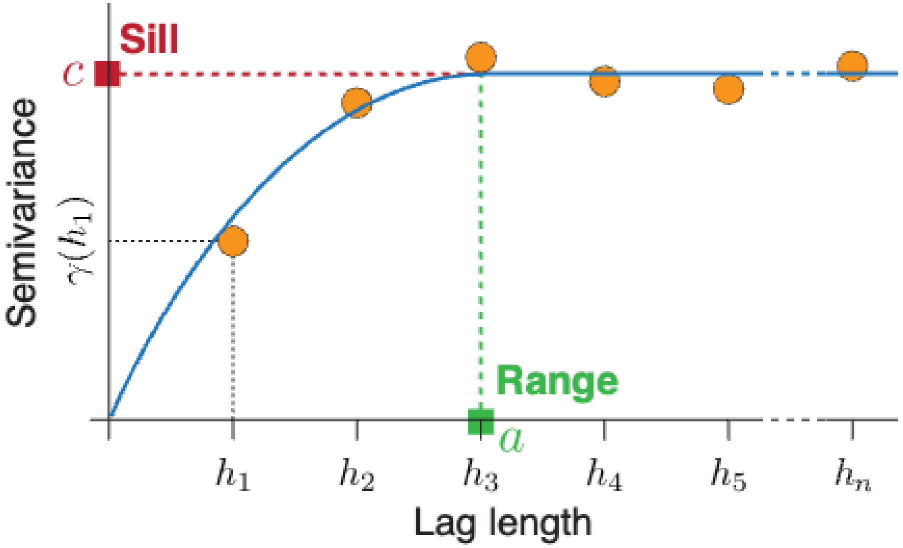
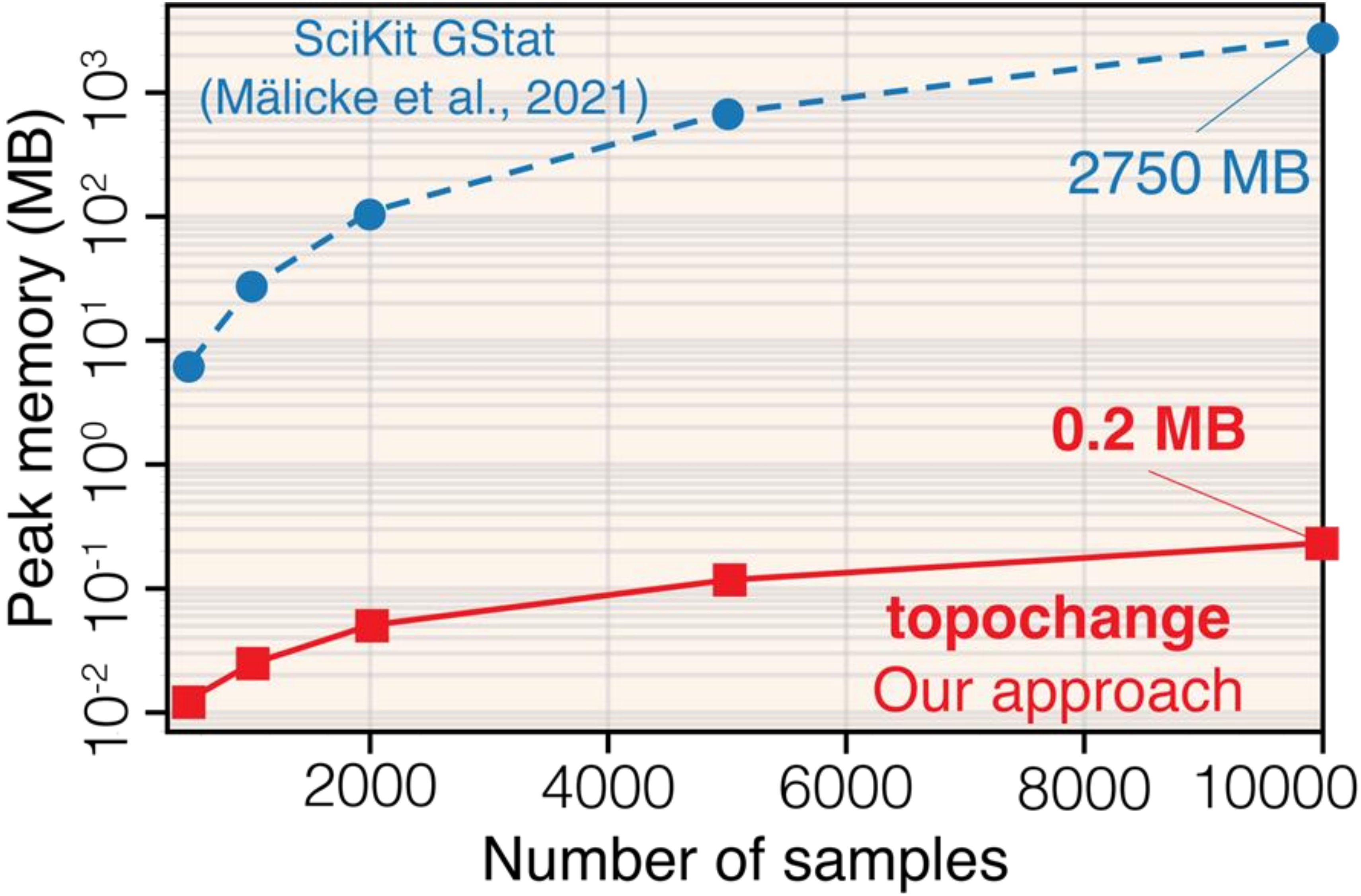


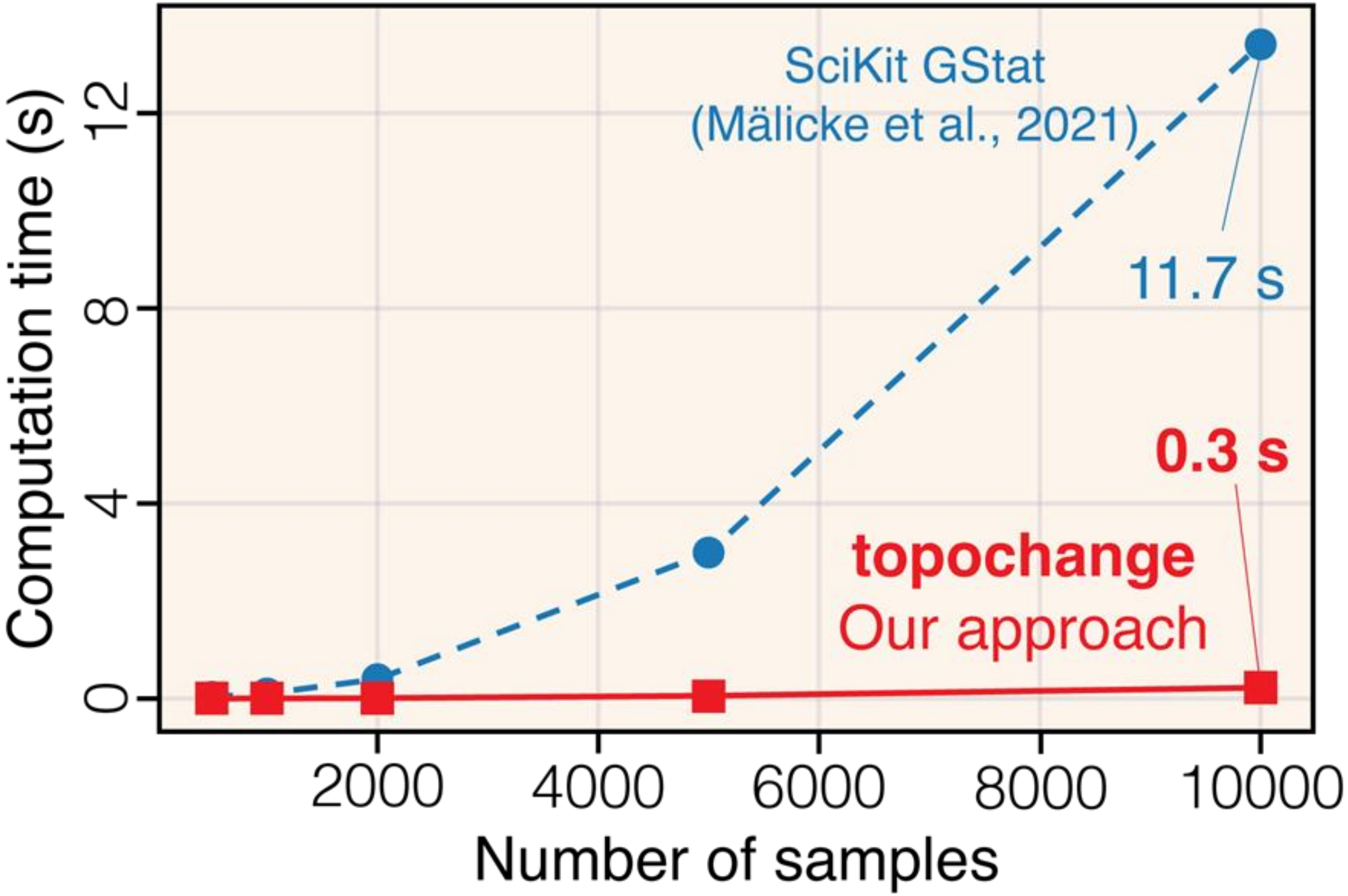
Computational innovations to allow wide-scale analysis



Variogram analysis has been limited to smaller areas due to memory requirements.

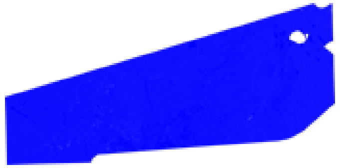
We fuse distance calculation, differencing, and bin accumulation into a single Numba-compiled parallel kernel with O(n_bins) memory (compared to the O(n_bins^2) approach of SciKit Gstat)

Computational innovations to allow wide-scale analysis



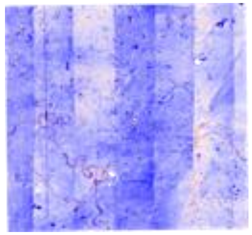
Benchmarking of the variogram computation in topochange against SciKit-GStat shows approximately 40× faster computation times (0.3 s vs. 11.7 s) and four orders of magnitude lower peak memory usage (0.2 MB vs. 2,750 MB) at 10,000 samples, enabling variogram estimation on large datasets without specialized hardware

Uncertainty in Vertical Differencing

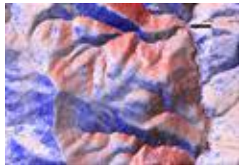


Very long range → **Vertical bias**

– 0.03 m



Long range



Mid range

Short range

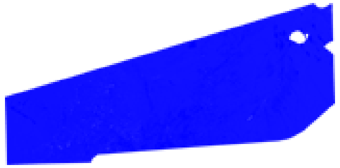
→ **Mean correlated uncertainty values over three length scales**

Very short range → **Mean uncorrelated uncertainty**

Uncertainty in Vertical Differencing

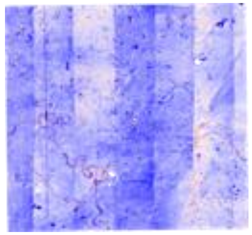
Very short range → Mean uncorrelated uncertainty

Uncertainty in Vertical Differencing

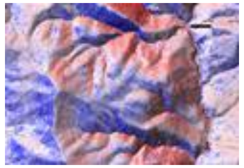


Very long range → **Vertical bias**

– 0.4 m



Long range



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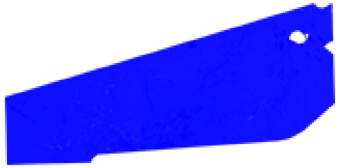
Uncertainty in Vertical Differencing



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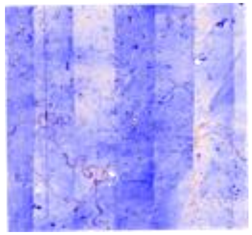
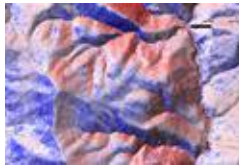
– 0.03 m





Long range

Mid range

Short range

→ **Mean correlated uncertainty values over three length scales**

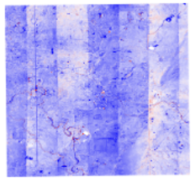
Very short range → **Mean uncorrelated uncertainty**

Total mean uncertainty

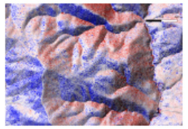
± 0.04 m

Questions?

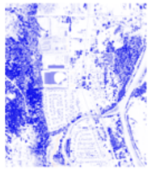
Very long range → **Vertical bias** – 0.03 m



Long range

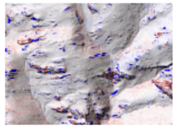


Mid range



Short range

→ **Mean correlated uncertainty values over three length scales**



Very short range → **Mean uncorrelated uncertainty**

Total mean uncertainty

± 0.04 m

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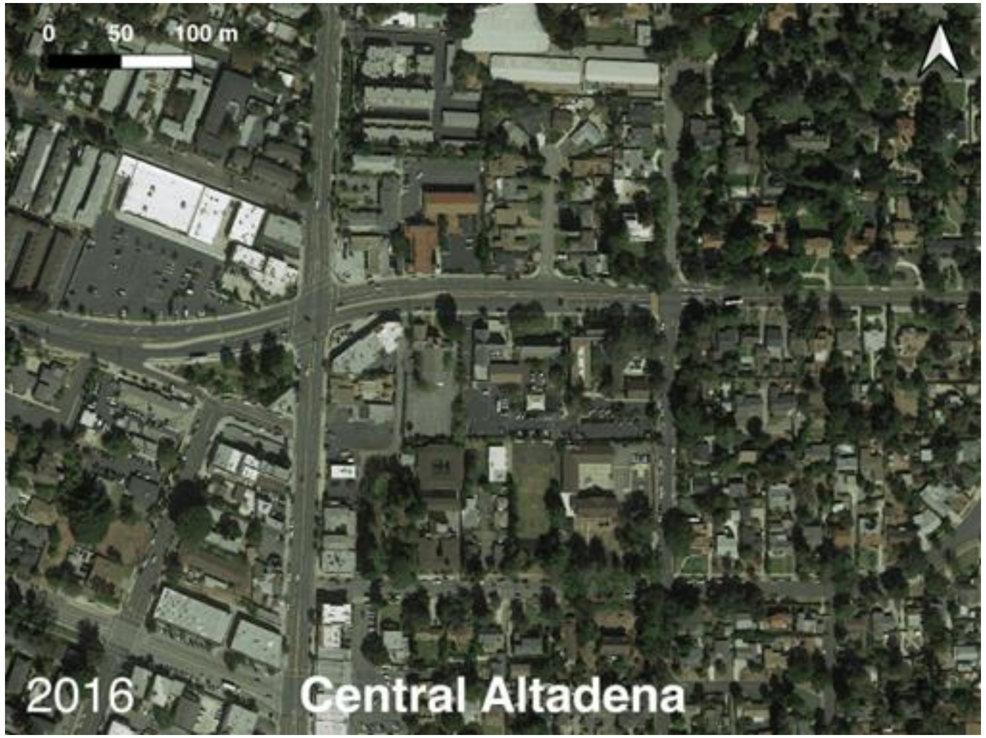

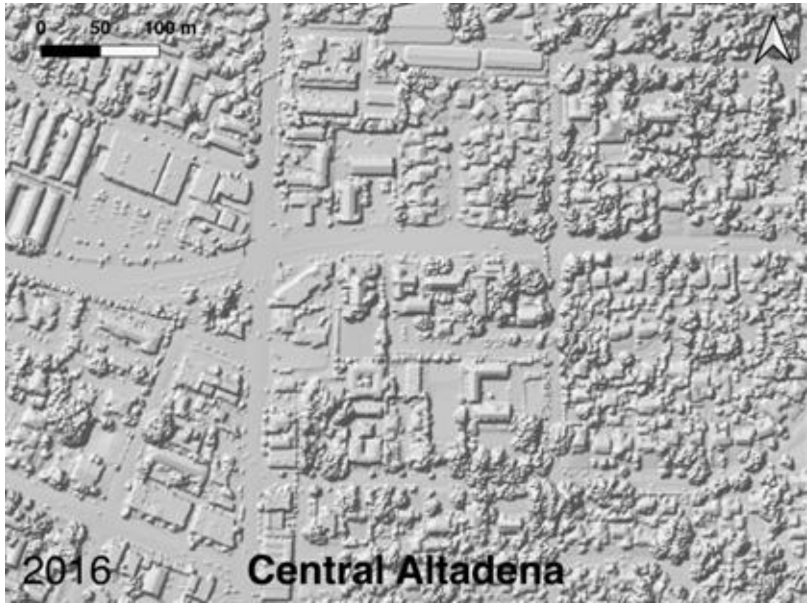

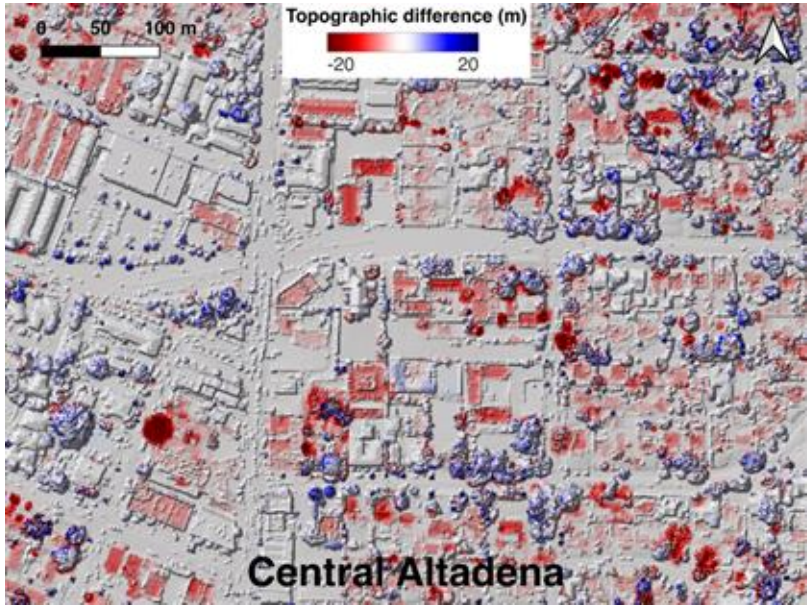

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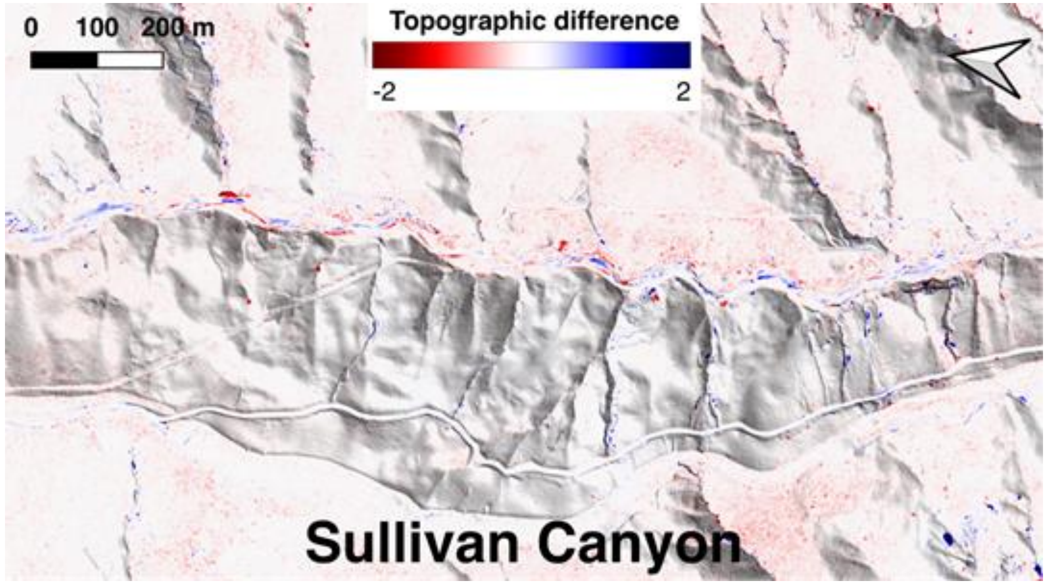

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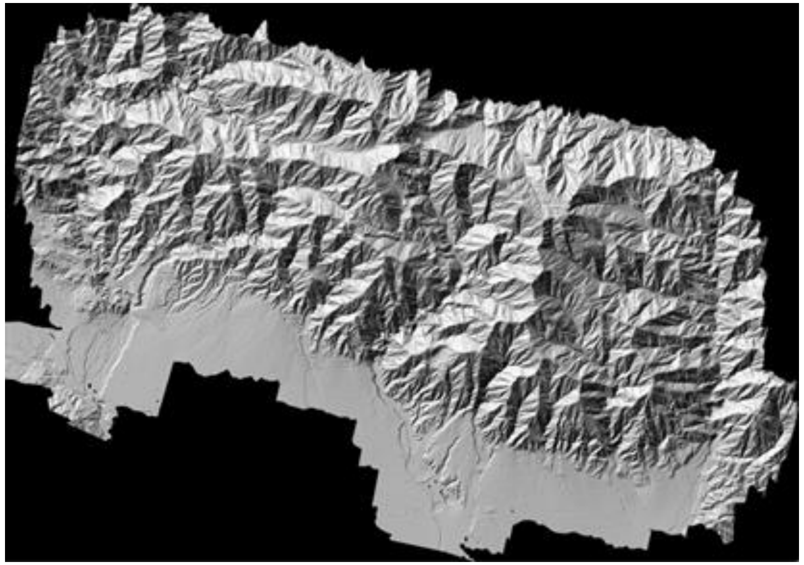

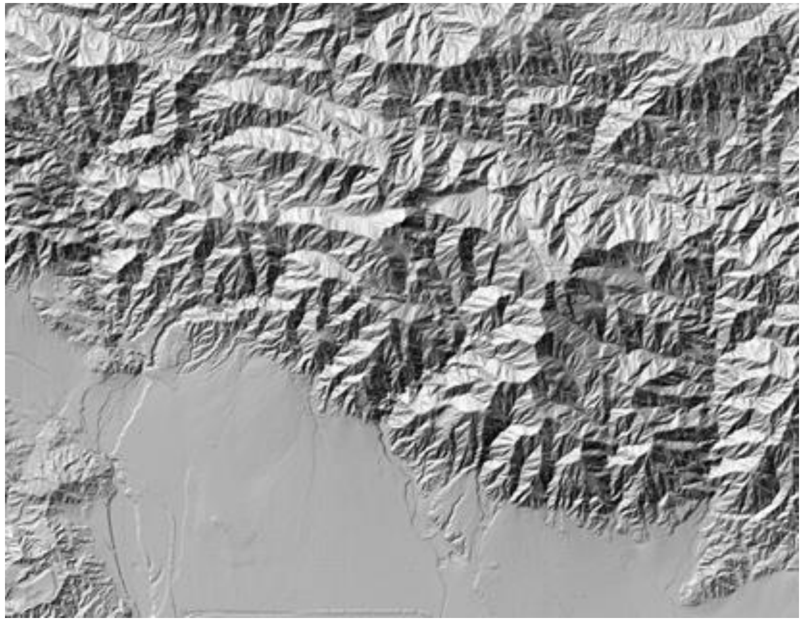

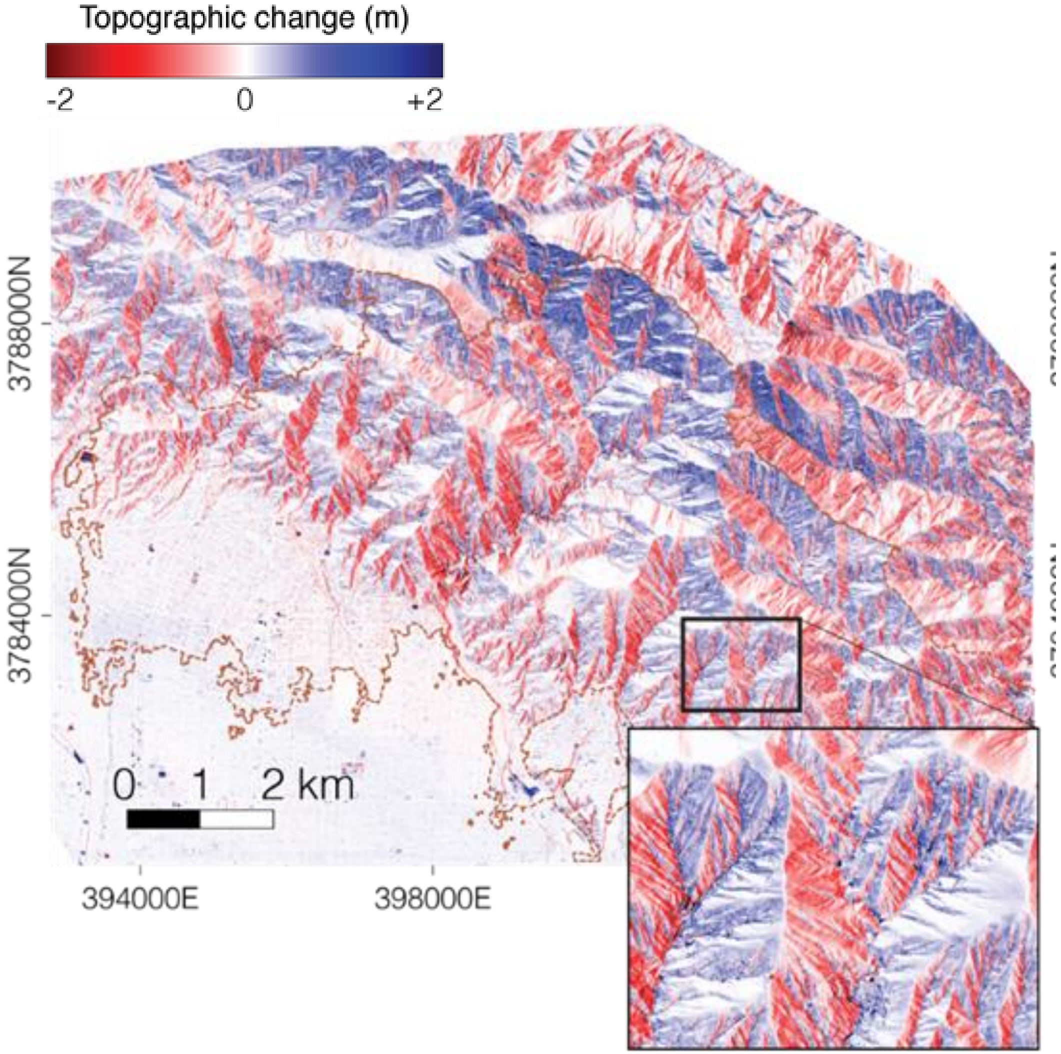

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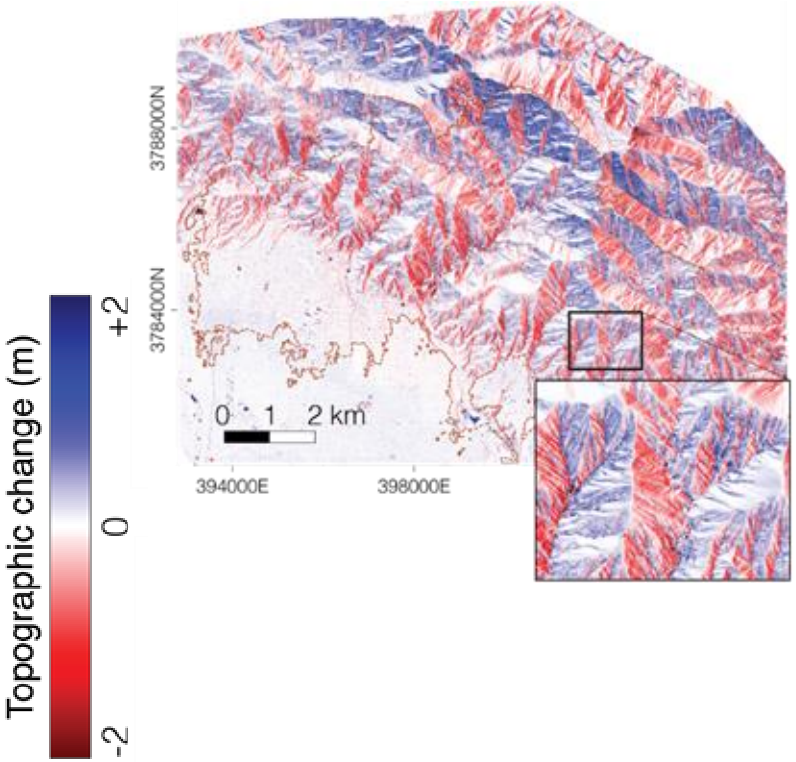

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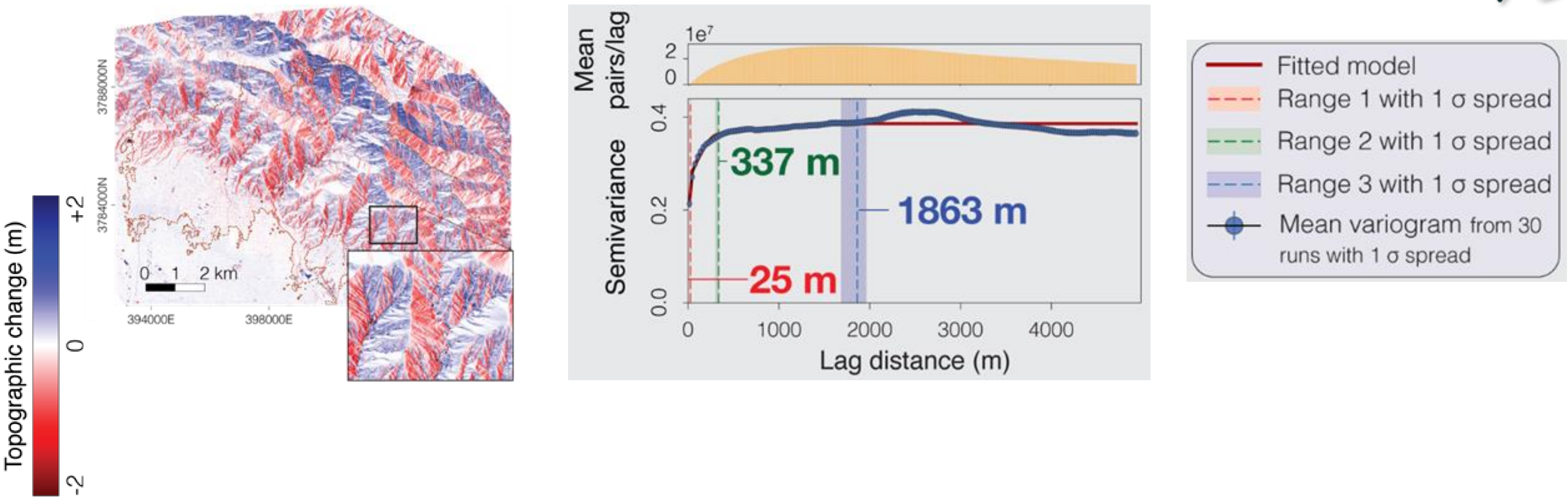

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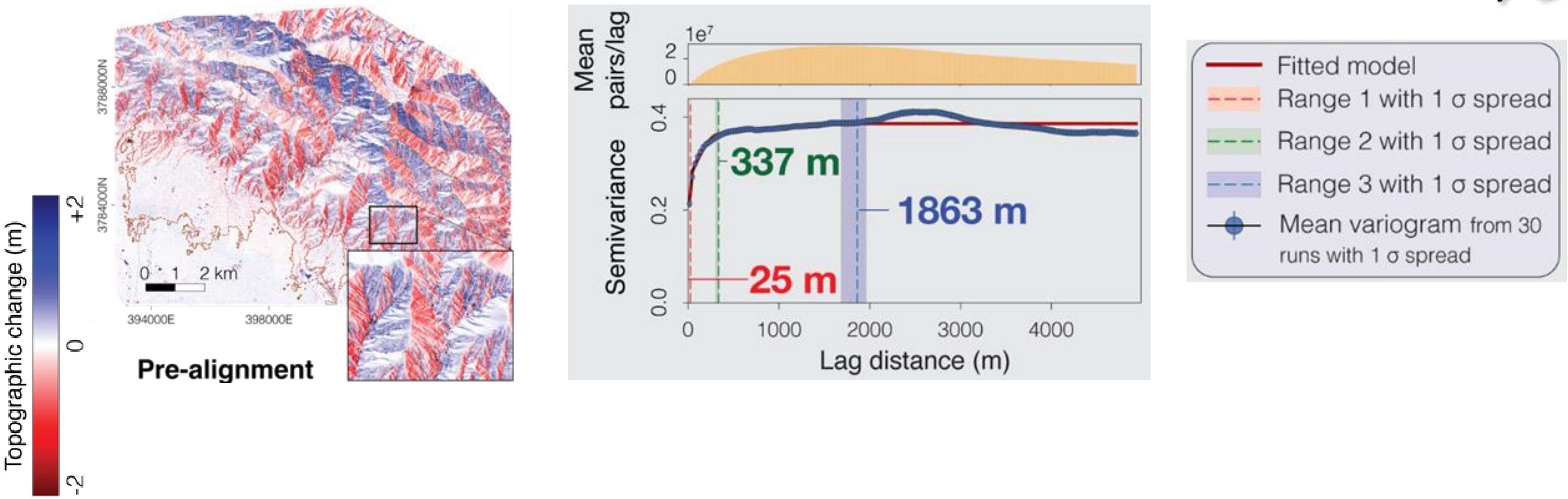

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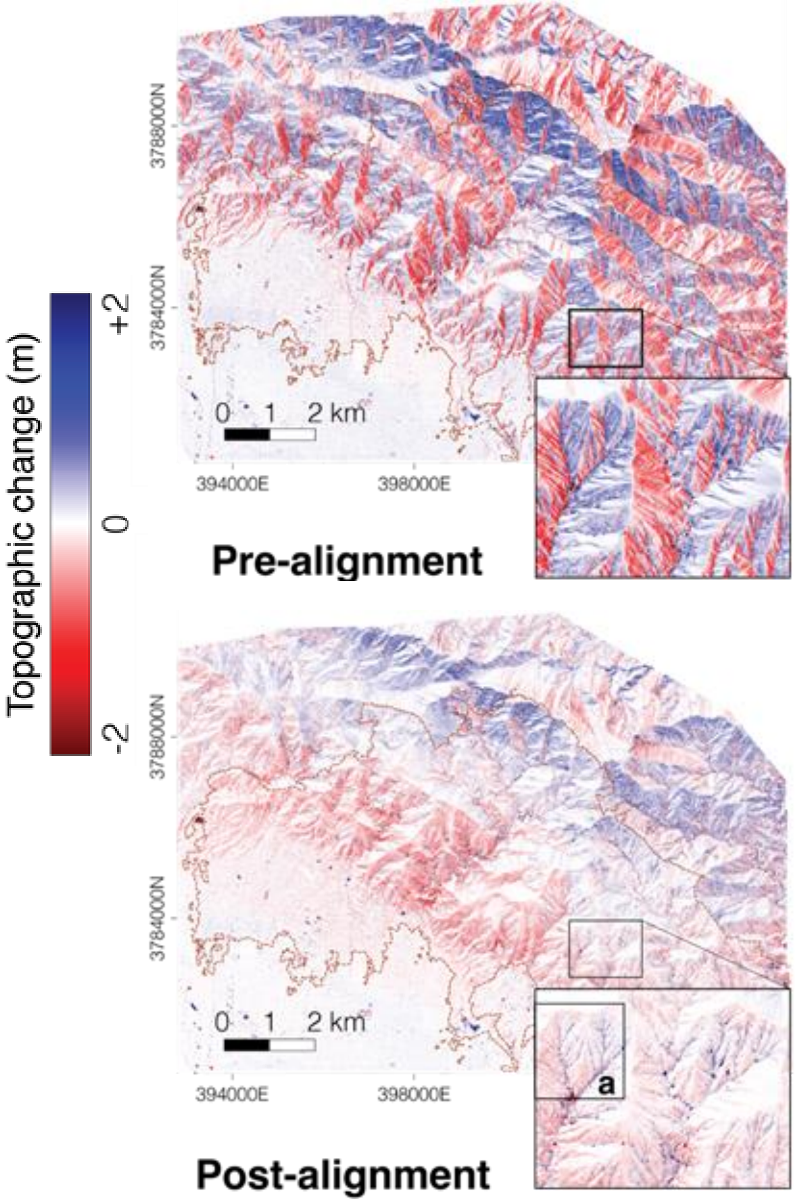

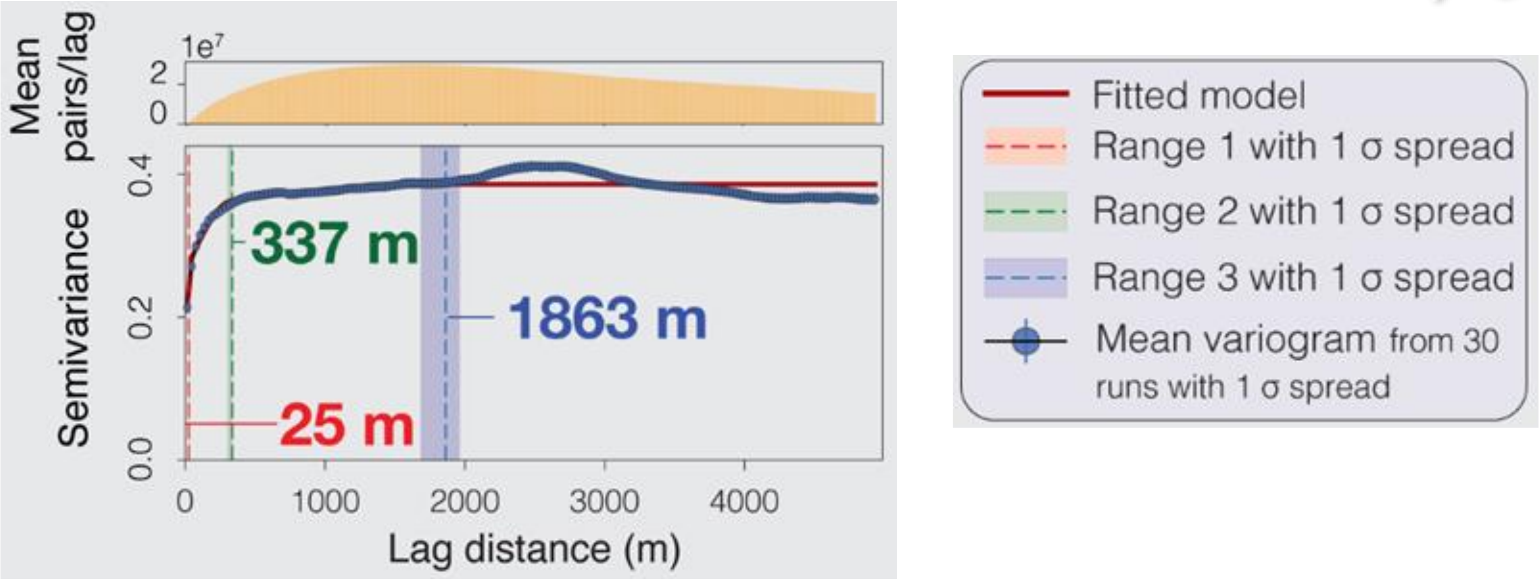

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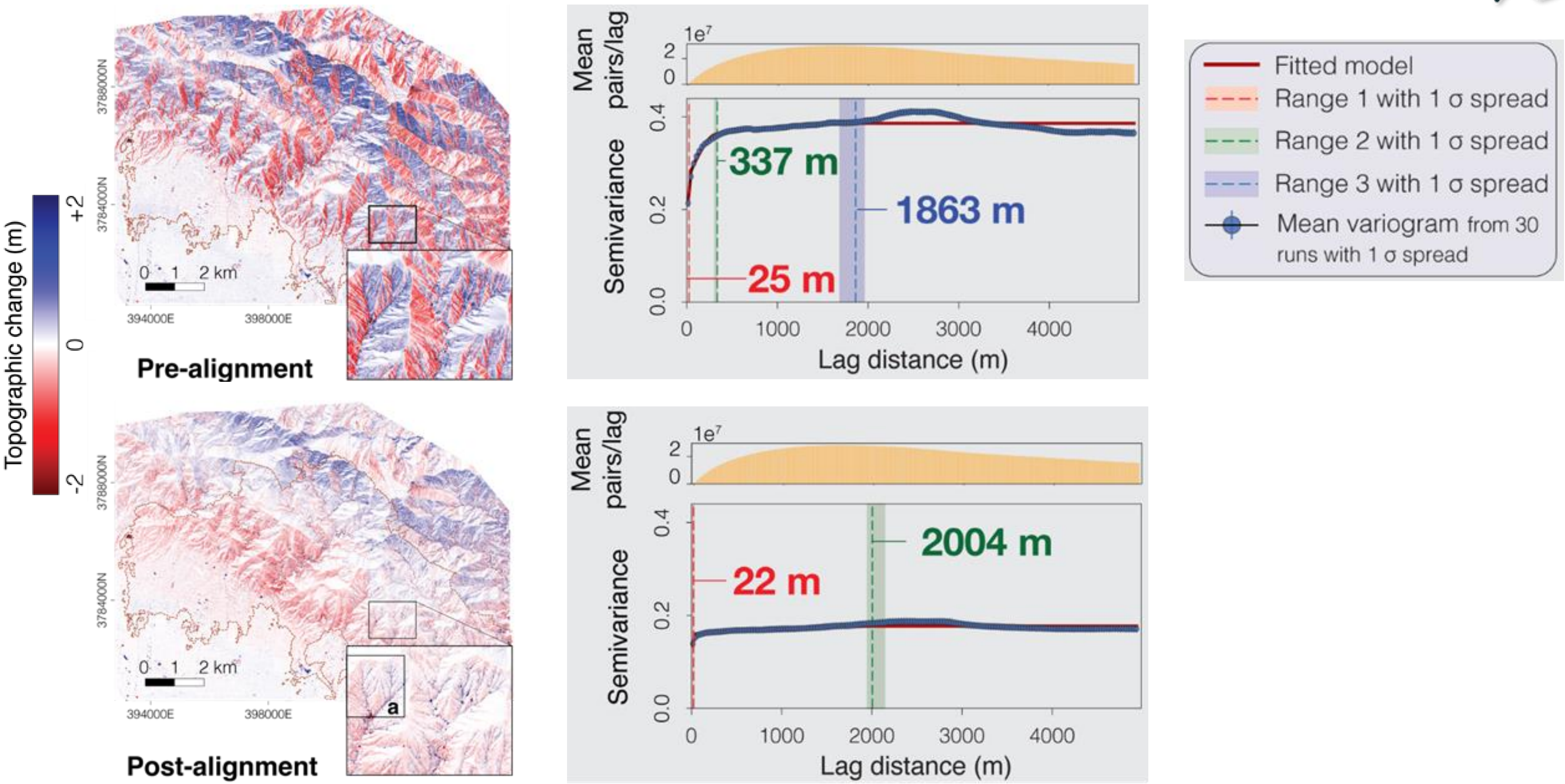

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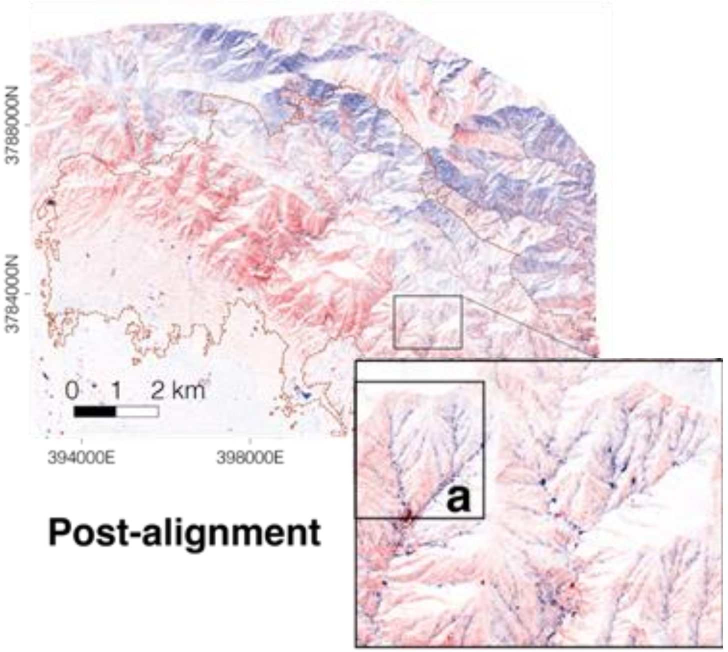
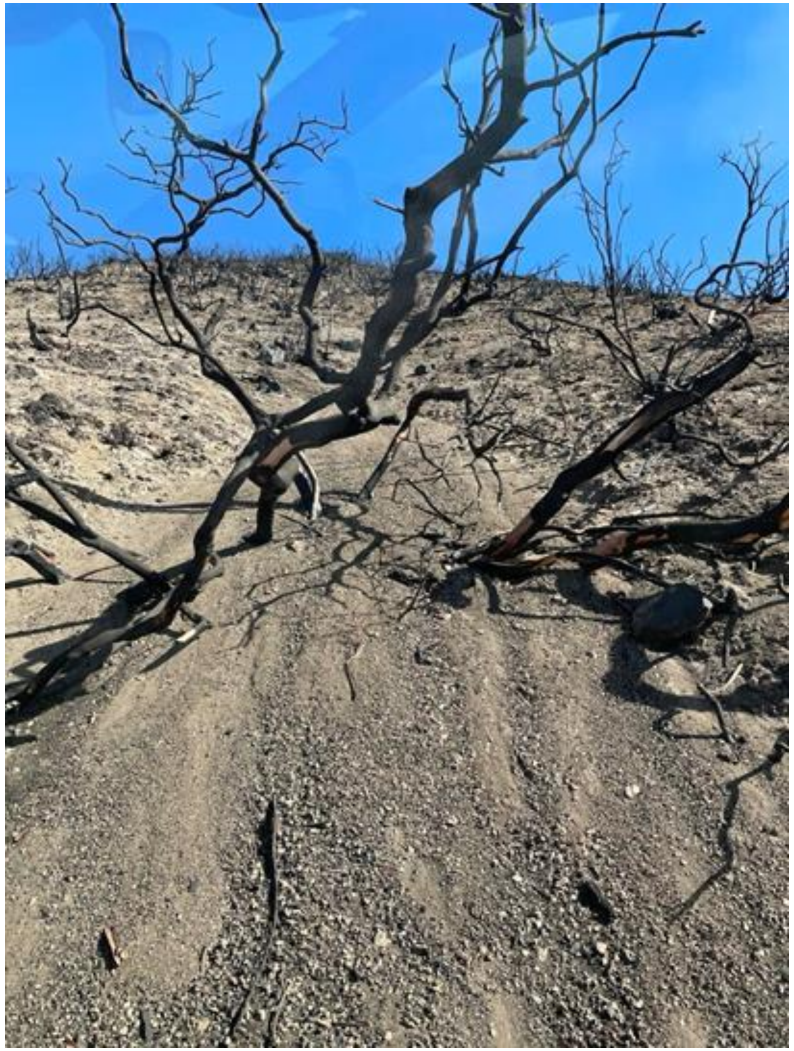
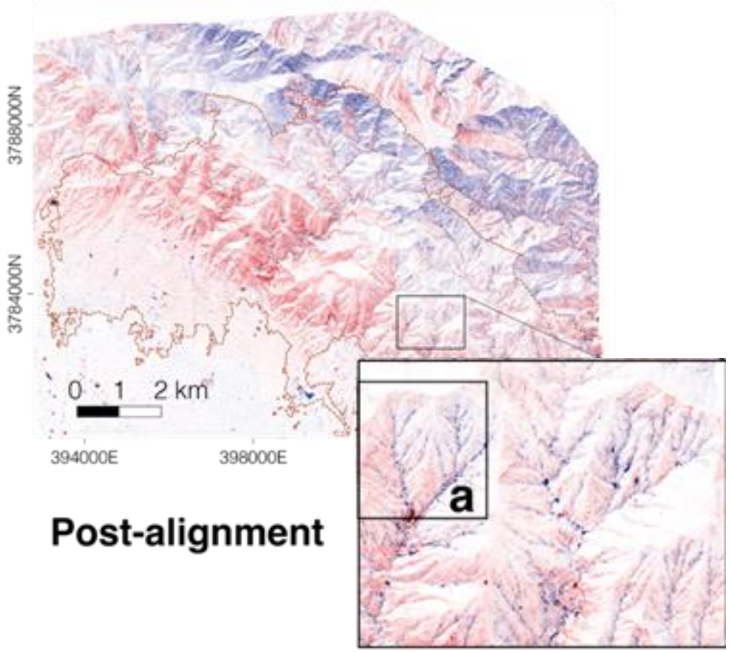
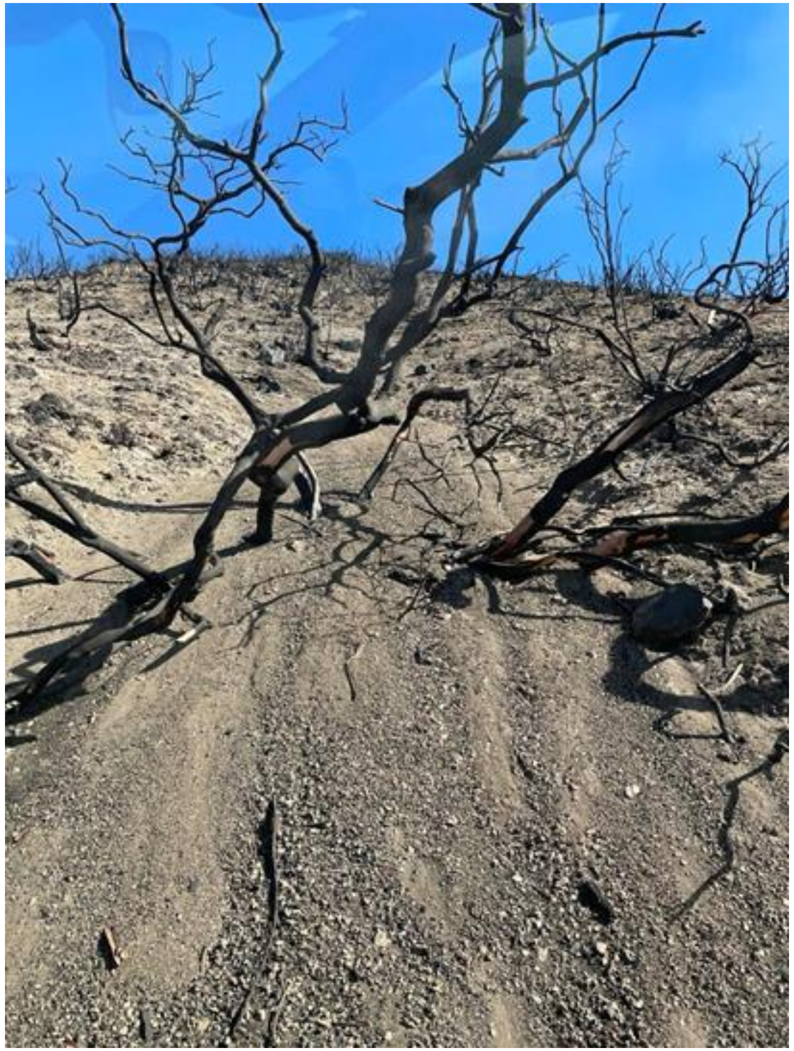


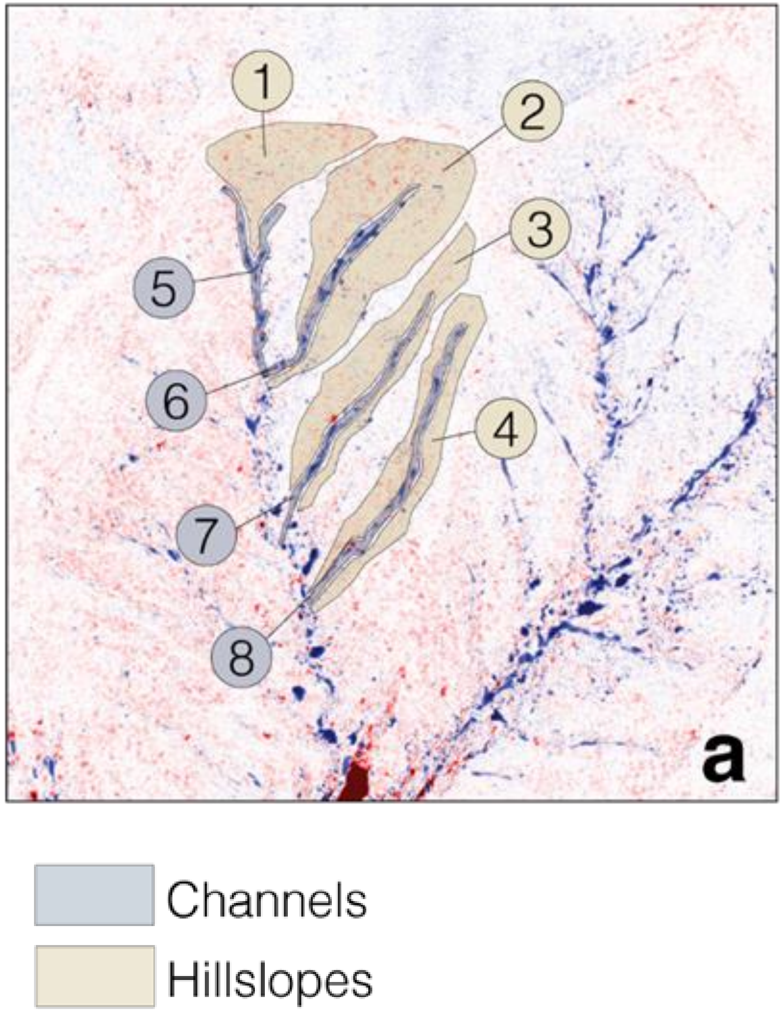
Post-alignment



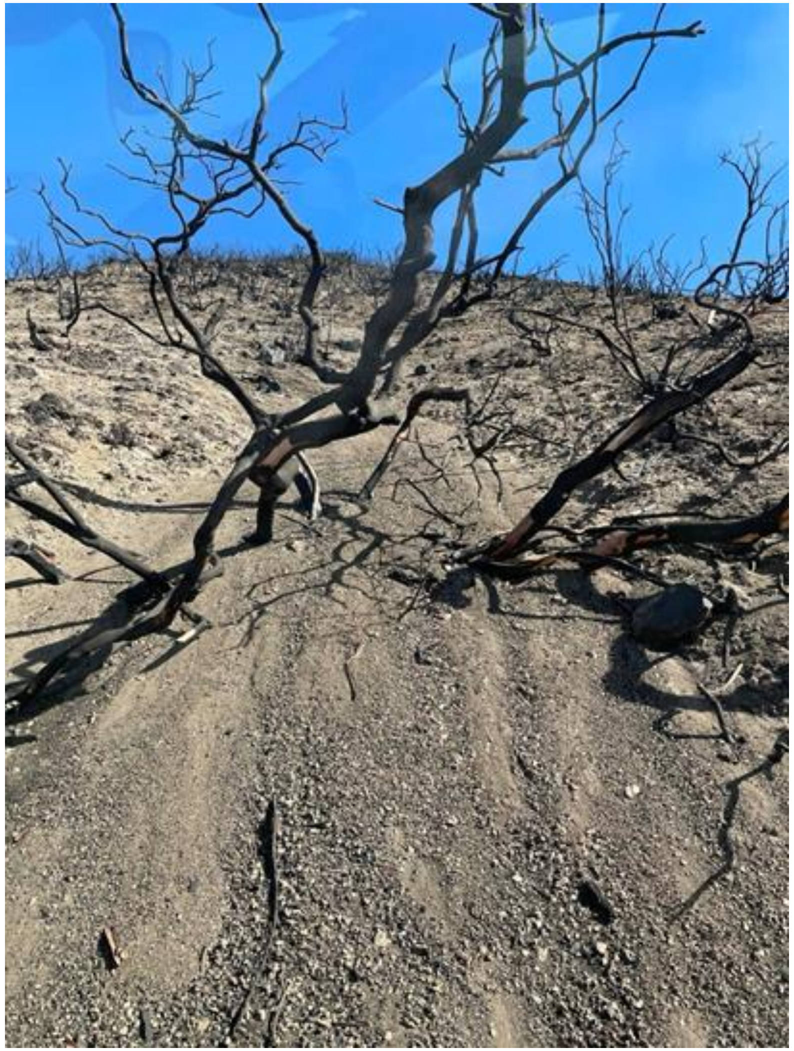
Application: 2025 Los Angeles Fires, CA

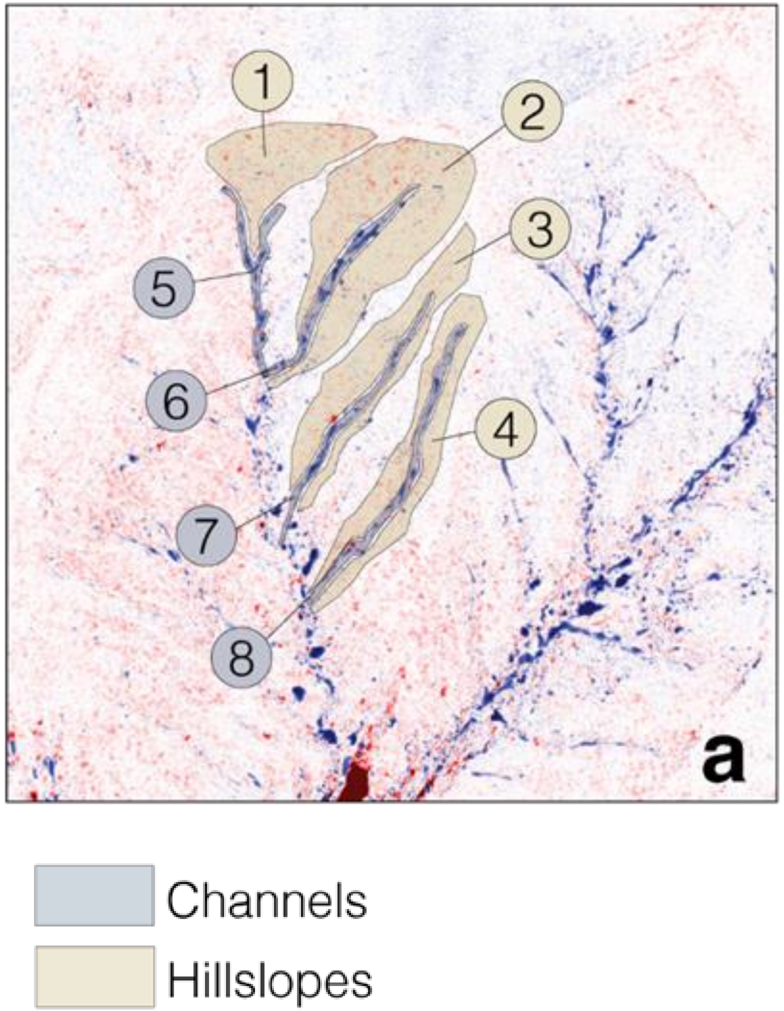






Application: 2025 Los Angeles Fires, CA





Net change with propagated uncertainties using post-alignment variogram model

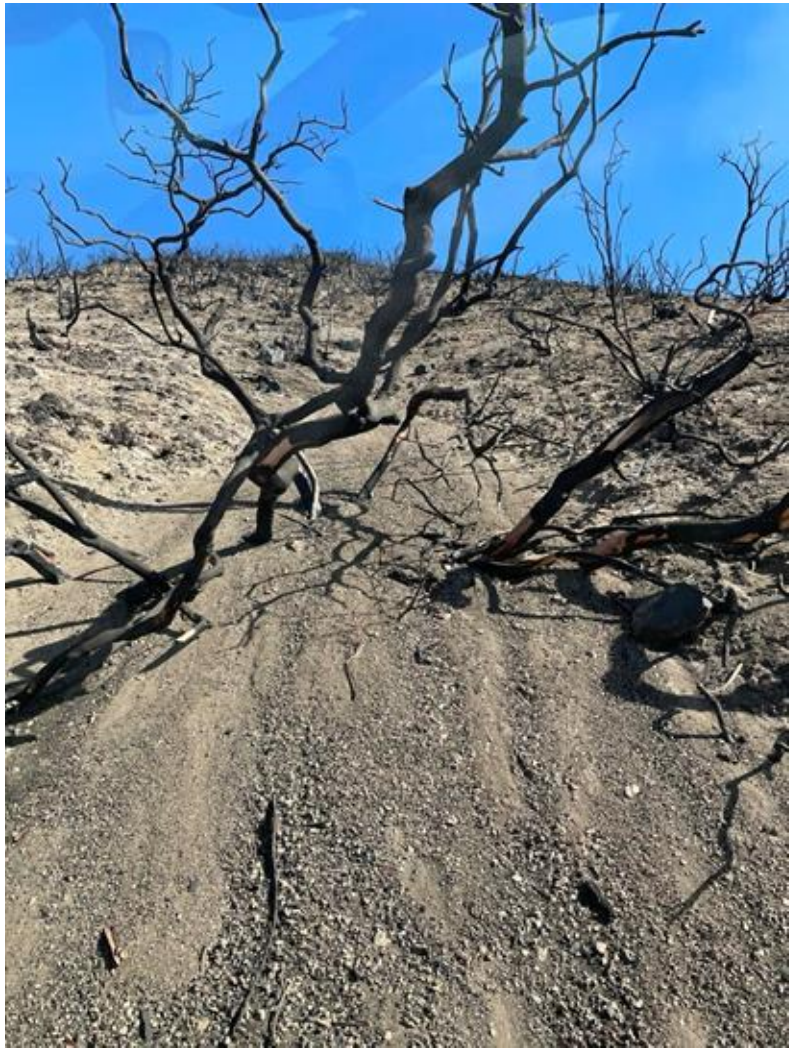
1 -0.053 **± 0.121 m**

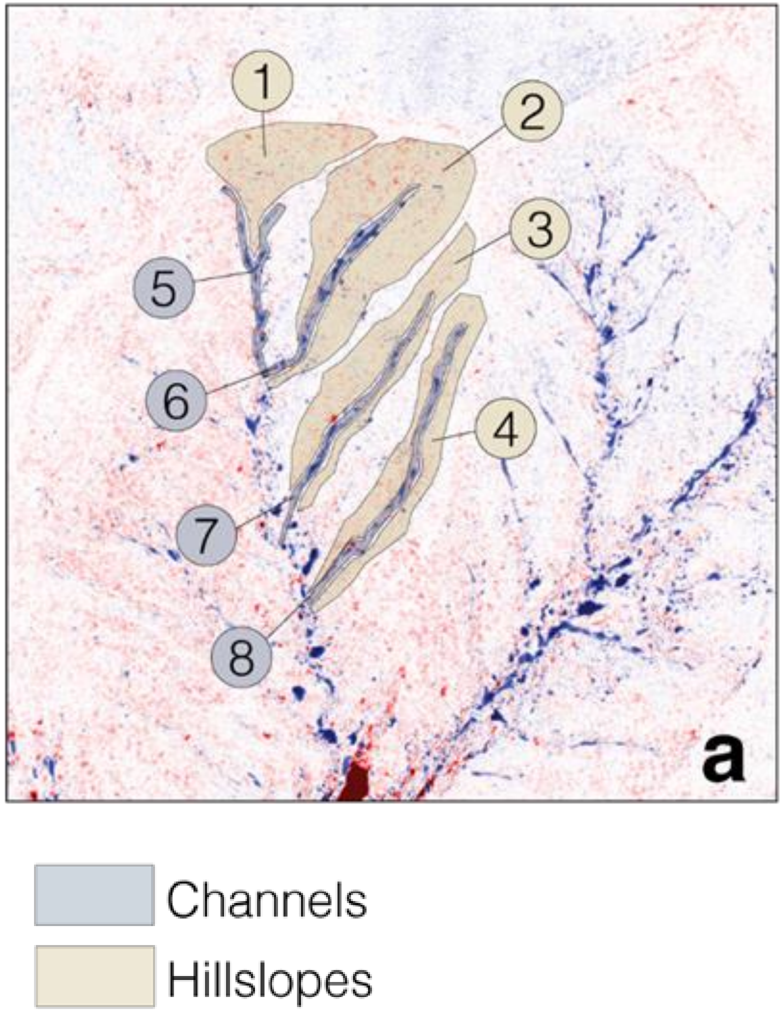
2 -0.018 **± 0.110 m**

3 -0.008 **± 0.114 m**

4 -0.024 **± 0.113 m**

Application: 2025 Los Angeles Fires, CA





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1. -0.053 ± **0.121 m**
2. -0.018 ± **0.110 m**
3. -0.008 ± **0.114 m**
4. -0.024 ± **0.113 m**
5. +0.226 ± **0.137 m**
6. +0.326 ± **0.138 m**
7. +0.232 ± **0.134 m**
8. +0.166 ± **0.130 m**

Uncertainty on OpenTopography

LABS

3. Error handling (optional)

- [x] Estimate mean uncertainty using geostatistical approach.

Draw the boundaries of areas where you want to calculate mean elevation change (e.g., landslides, glaciers, unstable terrain).
Because errors in nearby pixels are spatially correlated, the uncertainty in a mean topographic change value depends on the size and shape of the averaging area: smaller regions have proportionally higher uncertainty than larger ones. The tool uses your drawn polygons to calculate area-specific uncertainty estimates based on how errors correlate at the distances present within each region.

- [ ] User-defined Minimum Level of Detection

Uncertainty on OpenTopography

View all output variables

Understanding correlation patterns at different scales with variogram analysis :

- Range 1: **321.3358 m**

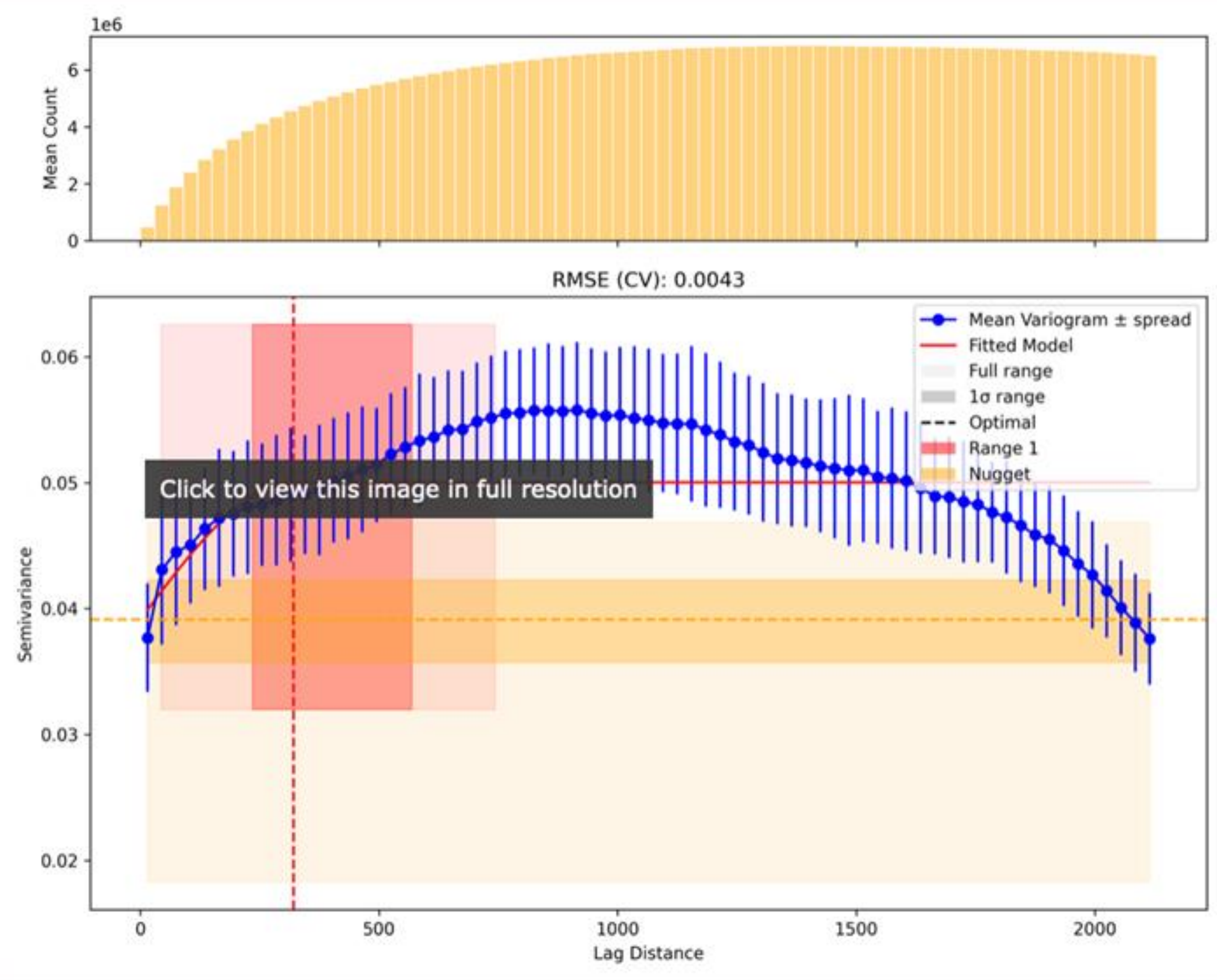
Nugget effect: 0.0391

Mean Uncertainty for feature(s) of interest:

- Polygon 1
  Total mean uncertainty: **±0.0270 m**
- Polygon 2
  Total mean uncertainty: **±0.0433 m**
- Polygon 3
  Total mean uncertainty: **±0.0279 m**

Mean uncertainty for entire area:

- Total mean uncertainty: **± 0.0088 m**



Uncertainty on OpenTopography

Polygon 1

Mean correlated uncertainty: ±0.0270 m
Mean correlated uncertainty (Min): ±0.0078 m
Mean correlated uncertainty (Max): ±0.0458 m
Uncorrelated uncertainty: ±0.0003 m

Total mean uncertainty: ±0.0270 m
Total mean uncertainty (Min): ±0.0078 m
Total mean uncertainty (Max): ±0.0458 m

Jupyter notebook for expert analysis

Differencing and geostatistical error analysis in lidar topographic differencing: Workflow starting with user provided point clouds

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1. Introduction

Vertical topographic differencing calculates net change in the vertical dimension by comparing digital elevation models (DEMs) collected at different times (Izumida et al., 2017; Wheaton et al., 2010). Topographic differencing underpins a wide range of studies, spanning vegetation biomass changes, lava-flow emplacement (Albino et al., 2015), fluvial and coastal floods (Izumida et al., 2017), landslides (Lucieer et al., 2014), fine-scale fluvial sediment budgets (Wheaton et al., 2010), and tectonic activity (Langridge et al., 2014; Scott et al., 2018).

Vertical topographic differencing

Vertical differencing is the pixel-by-pixel subtraction of two digital-elevation models (DEMs) that share a common coordinate system and identical grid geometry (e.g., Scott et al., 2021). Because every cell occupies the same planimetric position in both rasters, subtracting them yields a raster of elevation change (Δz). Researchers rarely inspect the differencing value of every cell individually. Instead, aggregate change is summarized over a specific feature or landform (net sediment deposition on a bar, net tree growth in a reforested stand, net inflation along a volcanic flank, etc.) The mean elevation change over a polygon Ω is:

$$\Delta z^{aggregate} = \frac{1}{N}\sum_{i=1}^{N}\Delta z_i$$

where the sum of i individual topographic change measurements (Δz_i) is taken over the N cells that fall inside Ω.

Why uncertainty matters

The elevation change observed in a differenced raster can be decomposed as:

$$\Delta_z^{measured} = \Delta_z^{actual} + \Delta_z^{error}$$

where:

- Δz_{actual} is the true vertical change (erosion, deposition, uplift, subsidence, construction, vegetation change, etc.)
- Δz_{error} is the cumulative vertical error from data collection, processing, and alignment (GNSS/INS errors, boresight errors, DEM interpolation artifacts, alignment errors, metadata errors)

Errors present in either dataset are often of similar magnitude to the true change, particularly for legacy datasets acquired with less advanced instrumentation, lower point density, or incomplete metadata (Glennie et al., 2014).

Error types in lidar topographic differencing

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Thank you!

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